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Arbitration Under Different Information

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1 Introduction

To model arbitration, it is assumed that each disputant has an estimate about the fair settlement of the arbitrator. Disputants may have a common estimate, as discussed in [2], or different estimates, as discussed in [1]. [2] presents a new arbitration mechanism double-offer arbitration (DOA) and analyzes DOA under the assumption that disputants hold the common estimate. The results show that DOA may lead to a convergence of offers. [1] gives an analysis of FOA under the assumption that disputants may have different estimates but the estimates are common knowledge. The main result shows that FOA is impossible to induce the convergence of offers. This paper intends to investigate DOA under a similar assumption of [1]. We introduce a fuzzy variable which describes the word "close", and show that it is possible that DOA induces the convergence of offers.

For notational convenience, in this paper, a dispute is described as to determine a price x for a transaction between a seller s and a buyer b. Of course, s hopes a high x and b hopes a low x. The arbitrator's fair settlement is denoted by z_a, which is not completely known by s and b. However, s (b) estimates z_a as a random variable with probability density function $f_s(\cdot)$ ($f_b(\cdot)$) on $(-\infty, \infty)$. Their distribution functions are $F_s(\cdot)$ and $F_b(\cdot)$ respectively. This paper supposes that $f_s(\cdot) \neq f_b(\cdot)$.

2 FOA under Different Information

At first, as in [1], we introduce an auxiliary function $\phi(x) = F_b(x)(1 - F_s(x))$. This paper assumes that ϕ is differentiable and it is maximized at a unique point m_0. Therefore, $\phi'(m_0) = 0$, hence $(1 - F_s(m_0))/f_s(m_0) = F_b(m_0)/f_b(m_0)$.

If two offers do not converge, then under FOA the expected payoffs to two disputants s and b are

$$U_s(x_s, x_b) = x_b F_s(\frac{x_s + x_b}{2}) + x_s(1 - F_s(\frac{x_s + x_b}{2})) - m_s$$

$$U_b(x_s, x_b) = m_b - x_b F_b(\frac{x_s + x_b}{2}) - x_s(1 - F_b(\frac{x_s + x_b}{2}))$$

Theorem 2.1 [1] *Suppose that $f_s(\cdot)$ and $f_b(\cdot)$ are differentiable at m_0, then*
1) *if*

$$\begin{cases} 2f_b^2(m_0) > F_b(m_0) f_b'(m_0) \\ 2f_s^2(m_0) > -(1 - F_s(m_0)) f_s'(m_0), \end{cases} \tag{2.1}$$

then

$$\begin{cases} x_s^{\sharp} = m_0 + \frac{1 - F_s(m_0)}{f_s(m_0)} \\ x_b^{\sharp} = m_0 - \frac{F_b(m_0)}{f_b(m_0)} \end{cases}$$

are local Nash equilibrium offers under FOA;
2) *the above local Nash equilibrium is global if and only if*

$$\begin{cases} \int_{m_0}^{t} f_s(x)dx \geq \frac{t - m_0}{t - x_b^{\sharp}}(1 - F_s(m_0)) & \text{for } t > x_b^{\sharp} \\ \int_{m_0}^{t} f_b(x)dx \leq \frac{t - m_0}{x_s^{\sharp} - t}(F_b(m_0)) & \text{for } t < x_s^{\sharp}. \end{cases}$$

Corollary 2.1 *The convergence of offers under FOA is impossible.*

3 DOA under Different Information

3.1 An Introduction of DOA

DOA separates disputants' estimates from their demands and let both disputants give double offers. In other words, two disputants s and b are required to give double offers (x_s, y_s) and (x_b, y_b), respectively, where x_s and x_b are called the primary offers while y_s and y_b are called the secondary offers. Based on these, the arbitrator, with fair settlement z_a, determines his settlement as described in the flowchart of Figure 1, where

$$C_s(x_s, y_s | z_a) = \alpha |y_s - x_s| + (1 - \alpha)(y_s - z_a)$$
$$C_b(x_b, y_b | z_a) = \alpha |y_b - x_b| + (1 - \alpha)(z_a - y_b)$$

are called the arbitrator's criterion functions for s and b respectively. Note that in the criterion functions, a parameter α expresses a weight on two terms, which is announced by the arbitrator before arbitration.

Let $(x_s^*, y_s^*, x_b^*, y_b^*)$ be possible Nash equilibrium offers of s and b under DOA.

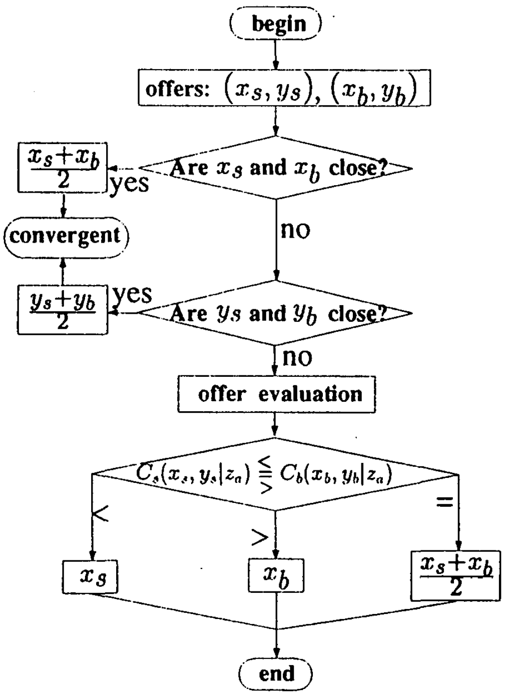

Figure 1: Flowchart of DOA

Lemma 3.1 $x_s^* \geq y_s^* \geq y_b^* \geq x_b^*$ *and* $x_s^* - x_b^* \geq x_s^\sharp - x_b^\sharp$.

In DOA of Figure 1, the word "close" is described by a fuzzy variable. In other words, distance $t \in [0, C]$ is thought to be "close" with probability $h(t)$, where $h(t) \in [0,1]$ for all $t \in [0, C]$. For convenience, we assume that

$$C < \frac{1 - F_s(m_0)}{f_s(m_0)} + \frac{F_b(m_0)}{f_b(m_0)} = x_s^\sharp - x_b^\sharp,$$

which says that the offers under FOA are not close. Furthermore, $h'(t) \leq 0$, since a shorter distance means closer. We also suppose that $h''(t)$ always exists.

Let

$$A = A(x_s, y_s, x_b, y_b) = \frac{(1-2\alpha)(y_s + y_b) + \alpha(x_s + x_b)}{2(1-\alpha)}.$$

By Lemma 3.1, the expected payoffs to players s and b are as follows:

$$\begin{aligned} U_s &= \left(\frac{y_s + y_b}{2} - m_s\right) h(y_s - y_b) \\ &\quad + (x_s - m_s)(1 - h(y_s - y_b)) \\ &\quad + (x_b - x_s) F_s(A(x_s, y_s, x_b, y_b))(1 - h(y_s - y_b)) \\ U_b &= \left(m_b - \frac{y_s + y_b}{2}\right) h(y_s - y_b) \\ &\quad + (m_b - x_s)(1 - h(y_s - y_b)) \\ &\quad + (x_s - x_b) F_b(A(x_s, y_s, x_b, y_b))(1 - h(y_s - y_b)) \end{aligned}$$

3.2 Nash Equilibrium Offers

Checking the first order conditions, we know that the primary offers are:

$$\begin{cases} x_s^* = m_x + \frac{1-F_s(m_0)}{f_s(m_0)} \frac{1-\alpha}{\alpha} \\ x_b^* = m_x - \frac{F_b(m_0)}{f_b(m_0)} \frac{1-\alpha}{\alpha}, \end{cases} \quad (3.1)$$

where m_x satisfies

$$m_0 = \frac{(1-2\alpha)(y_s + y_b) + 2\alpha m_x}{2(1-\alpha)}.$$

On the other hand, if $f_s(m_0) = f_b(m_0)$, then the first order conditions with respect to y require that the secondary offers are $y_s^* = m_y + u$, $y_b^* = m_y - u$, where

$$m_y = m_x = m_0,$$

and u satisfies

$$\begin{aligned} &\tfrac{1}{2} h(2u) - \tfrac{1-2\alpha}{\alpha}(1 - F_s(m_0))(1 - h(2u)) \\ &+ (2F_s(m_0) - 1) \tfrac{1-\alpha}{\alpha} \tfrac{1-F_s(m_0)}{f_s(m_0)} h'(2u) = 0. \end{aligned} \quad (3.2)$$

By further checking the second order conditions, we have

Theorem 3.1 *If* $h''(t) \geq 0$, $f_s(m_0) = f_b(m_0)$ *and* (2.1) *holds then* $(x_s^*, y_s^*, x_b^*, y_b^*)$ *satisfying* (3.1) *and* (3.2) *form a local Nash equilibrium.*

Example 1: $F_s(x)$ and $F_b(x)$ are uniform distributions on $[0,1]$ and $[-1/2, 1/2]$ respectively, $\alpha = 1/3$. Then $m_x = m_y = m_0 = 1/4$, $x_s^* = 7/4$, $x_b^* = -5/4$. If

$$h(t) = \begin{cases} \frac{e}{e-1} e^{-t} - \frac{1}{e-1} & \text{if } t \in [0,1] \\ 0 & \text{otherwise,} \end{cases}$$

then $h''(t) \geq 0$. Furthermore, the solution of (3.2) is $u = 0.235$ therefore $y_s^* = 0.485$, $y_b^* = 0.015$. Since $y_s^* - y_b^* = 0.47$, this distance is thought to be close with probability $h(0.47) \doteq 0.4 > 0$.

In the above example, $\alpha = 1/3$. If $\alpha \to 0$, then the probability that secondary offers are thought to be close becomes high. However, according to (3.1), two primary offers are more divergent.

参考文献

[1] Guo W.-G. and Chen T. (1995): "Final-offer arbitration with incomplete information", *Control and Decision*, 10, No. 1, 40-44 (In Chinese).

[2] Zeng D.-Z., Nakamura S. and Ibaraki T. (1995): "Double-offer arbitration", to appear in *Mathematical Social Sciences*.