COMPETITIVE PREDICTION OF A RANDOM VARIABLE

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ABSTRACT Two players want to guess a realization of a random variable whose distribution is apriori known to both players. Each player chooses a moment. The winner is a player who has chosen the moment later than that chosen by his opponent but earlier than the realization of the random variable. Two models (*i.e.* Common and Each) of this competitive prediction problem are formulated and solved. Some extended models are also discussed. It is shown that there are several unsolved problems within or around this field.

1. Introduction and Summary

2. Competitive Prediction of a Common r.v.

I and II predict a r.v. $\tau \sim U_{[0,1]}$. Let x and y be players' predictions. Denoting by $s(f)$ the event $x<(>)\tau$ for I, and similarly for II also, the payoff is given by:

$$(*)\ \text{When} \begin{Bmatrix} f\text{-}f \\ f\text{-}s \\ s\text{-}f \\ s\text{-}s \end{Bmatrix} \text{happens, the game is} \begin{Bmatrix} \text{draw} \\ \text{II's win} \\ \text{I's win} \\ \text{I(II)'s win, if } x>(<)y;\ \text{draw, if } x=y \end{Bmatrix}$$

i.e. when s-s happens, the bolder player wins.

I gets from II, +1(−1, 0) if the game is I's win (II's win, draw), so that the payoff function is

$$(2.1)\qquad K(x,y)=\begin{cases} -x+2y-1, & \text{if } x<y \\ 0, & \quad = \\ -2x+y+1, & \quad > \end{cases}$$

Theorem 1. *For "competitive prediction of common r.v." with payoff function* (2,1), *the solution is: The common optimal strategy is*

$$f^*(x)=g^*(x)=\begin{cases} (1/2)(1-x)^{-3/2}, & 0\leqq x\leqq 3/4 \\ 0, & \text{elsewhere} \end{cases}$$

The value of the game is 0.

3. Competitive Prediction of Each r.v.

Let τ_1 and τ_2 be *iid* r.v.s with common distribution $U_{[0,1]}$ (This doesn't seem to lose generality, provided *iid* is assumed) Let x and y be players' predictions. Denoting, for example, a combination of I's success & II's failure (*i.e.* $x<\tau_1$ and $y>\tau_2$) by s-f, the payoff is given by (*) with the fourth line replaced by

(**) When s-s happens, the game is $\begin{Bmatrix} \text{highest predictor's win} \\ \text{I(II)'s win, if } \tau_1 <(>) \tau_2 \end{Bmatrix}$, if $x \begin{Bmatrix} \neq \\ = \end{Bmatrix} y$.

Therefore the expected payoff is

$$(3.2) \qquad K(x,y) = \begin{cases} -xy+2y-1, & \text{if } x < y \\ 0 & \quad = \\ xy-2x+1, & \quad > . \end{cases}$$

Theorem 2. *For "competitive prediction of each r.v." with payoff ft.* (3. 2), *the solution is: The common optimal strategy is*

$$f^*(x) = g^*(x) = \begin{cases} (1/4)(1-x)^{-3}, & 0 \leqq x \leqq 2/3 \\ 0, & \text{elsewhere} \end{cases}$$

The value of the game is 0.

Example 3. Let (τ_1, τ_2) be distributed according to bivariate uniform with pdf

$$h(t_1, t_2) = 1 + \gamma(1-2t_1)(1-2t_2), \quad (t_1, t_2) \in [0,1]^2, \ |\gamma| \leqq 1.$$

Then the payoff function becomes

$$K(x,y) = \begin{cases} -xy+2y-1-\gamma x\bar{x}y\bar{y}, & \text{if } x<y \\ 0 & \quad = \\ xy-2x+1+\gamma x\bar{x}y\bar{y}, & \quad > . \end{cases}$$

4. Non-Zero-Sum Games. (略)

5. Two Diversions (略)

5a. Case where Re-prediction is Allowed.

5b. Case where Prediction is Noisy

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