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Shortest Path Problem with Fuzzy Arc Length

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1 Introduction

We discuss the problem of finding the shortest paths from a fixed origin to a specified node in a network with arcs represented as fuzzy numbers. While proposing an algorithm for solving the problem, we are faced with ranking methods between fuzzy numbers, which have been proposed in numerous literatures.

Some algorithms [2–4] for solving the problem have been proposed. It is, however, difficult to apply another ranking methods to the algorithms owing to the base of a specific ranking method.

We propose a common algorithm for solving fuzzy shortest path problem, in which most type of ranking method between fuzzy numbers are available according to a decision maker's preference.

By the way, there exist two main approaches to non-fuzzy version of this problem, that is, network simplex method and dynamic programming. In the latter approach, some standard algorithms have been proposed by Dijkstra [1], Bellman and Ford and Floyd and Warshall. In this research, we expand the algorithm based on Dijkstra's method to fuzzy version, because it is solvable within $O(n^2)$ in worst case and is at ease of calculation and expansion.

The algorithm leads to some paths with minimal length, and then the alternative of them may be chosen according to the decision maker's preference.

2 Preliminaries

Before proposing the subsequent problem and algorithm, several definitions will be presented. A fuzzy number $\tilde{a}$ is a convex fuzzy subset of the real line $\mathcal{R}$ with a normalized membership function $\mu_{\tilde{a}}$.

Various methods for ranking two or more fuzzy numbers have been reported in the literature. Most of these ranking methods can be classified into three subclasses with the following properties : (a) total order relation, (b) partial order relation and (c) the other relation.

The former two are used for getting minimum or minimal from among several fuzzy numbers on the proposed algorithm. Some of these ranking methods used here will be introduced as follows : In the literature [5], four indices $\mathrm{PD}(\tilde{a}_i)$, $\mathrm{PSD}(\tilde{a}_i)$, $\mathrm{ND}(\tilde{a}_i)$ and $\mathrm{NSD}(\tilde{a}_i)$ are proposed, which mean grades of dominance of of a fuzzy number $\tilde{a}_i$ over all other $\tilde{a}_j$, $j \neq i$. These indices lead to getting a maximum or minimum among fuzzy numbers.

In the literature [6], the partial order relation between two fuzzy numbers, which is related to the both side of the membership function mutually, is defined.

3 Fuzzy Shortest Path Problem

Now we propose the shortest path problem with fuzzy arcs length (Fuzzy-SPP) as follows: Let $G = (\mathcal{N},\mathcal{A})$ be a network, where $\mathcal{N}$ is a set of nodes with n elements, and $\mathcal{A}$ is a set of arcs with length represented as fuzzy number.

The problem is to find the some paths with the minimal total length represented as fuzzy number from the origin to the destination on the network G. It is assumed that there exists the path from the origin to each node in the network.

On the basis of Dijkstra's method [1], we propose an algorithm for solving the Fuzzy-SPP.

First, we have to define the terminologies and variables used here.

$\widetilde{d}_{ij}$: a length represented as a non-negative fuzzy number between adjacent nodes i and j. If nodes i and j are not adjacent, $\widetilde{d}_{ij}$ implies $+\infty$.

U_j : a set composed of minimal fuzzy lengths from the origin node 1 to a node j,

minimal$\{S\}$: an operator to derive some minimal elements from the set S by using the order relation proposed in various literatures mentioned in section 2. It is noted that the operator "minimal" can be substituted by "minimum" if the relation is total order.

n : a number of nodes,

P : a set of nodes with permanent labels,

T : a set of nodes with temporary labels.

The algorithm is proposed as follows:

[step 0] Choose an order relation between fuzzy numbers according to a decision maker's preference. And give the values of parameters or thresholds if necessary.

[step 1] The sets U_j, P and T are initialized as follows:

$$U_j := \{D_{1j}\} \quad \text{for } j = 1, \ldots, n,$$

$$P := \{1\} \ , T := \{2, \ldots, n\}.$$

[step 2] If $T = \phi$, then go to step 4. Otherwise, find $j^* \in T$ such that

$$\text{minimal}\{\cup_{j \in T} U_j\} \cap U_{j^*} \neq \phi.$$

It is noted that j^* is uniquely determined when using total order relation, however, it may not be so in partial order. In this case, j with the smallest value among U_j, which satisfies above formula, is assigned to j^* alternatively.

Reset $T := T \setminus \{j^*\},\ P := P \cup \{j^*\}$. Go to step 3.

[step 3] For all $j \in T$, $U_j :=$ minimal$\{U_j \cup \{\widetilde{u}_k + \widetilde{d}_{j^*j} \mid \widetilde{u}_k \in U_{j^*}\}\}$, return to step 2.

[step 4] If a lot of paths with minimal length are obtained, the alternative of them should be chose according to the decision maker's preference, and then terminate. And if he/her does not satisfy the obtained path, return to step 0 and re-calculate with another alternative ranking method.

4 Conclusion

We proposed a common algorithm for solving fuzzy shortest path problem independent of the type of ranking method between fuzzy numbers. Some numerical examples will be presented at the conference.

References

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