1996年度日本オペレーションズ・リサーチ学会
秋季研究発表会

Bicriterion Shortest Path Problems According to a Decision Maker's Preference

01011600 Ashikaga Junior College OKADA Shinkoh

1 Introduction

The bicriterion shortest path problem (BSP) can be described over a directed network $G(N,A)$, consisting of a finite set N of n nodes and a finite set A of m directed arcs. Each arc is defined in terms of an ordered pair (i,j), where i and j denote the starting node and the ending node respectively. In bicriterion shortest path problem a vector $\boldsymbol{c}_{ij}=(c_{ij}^1,c_{ij}^2)$ such as 'cost', 'distance', 'duration time' and so on are associated with each arc(i,j). c_{ij}^1 and c_{ij}^2 are assumed to be positive.

In this network, we specify two nodes, denoted by s and t, which are the source node and the destination node, respectively. We define a path p_{ij} as a sequence $p_{ij}=\{i=i_1,(i_1,i_2),i_2,\ldots,i_{l-1},(i_{l-1},i_l),i_l=j\}$ of alternating nodes and arcs. The existence of at least one path p_{si} in $G(N,A)$ is assumed for every node $i\in N-\{s\}$. The distance vector $\boldsymbol{d}(p)$ along the path p is defined as $\boldsymbol{d}(p)=(d^1(p),d^2(p))=(\sum_{(i,j)\in p}c_{ij}^1,\sum_{(i,j)\in p}c_{ij}^2)$.

The problem BSP may be stated as the following bi-objective linear programming problem : P1

$$\min\ c^1(x)=\sum_{(i,j)\in A}c_{ij}^1x_{ij} \tag{1}$$

$$\min\ c^2(x)=\sum_{(i,j)\in A}c_{ij}^2x_{ij} \tag{2}$$

subject to

$$\sum_j x_{ij}-\sum_j x_{ji}=\begin{cases}1 & \text{if } i=s,\\ 0 & \text{if } i\neq s,t\\ -1 & \text{if } i=t.\end{cases} \tag{3}$$

$$x_{ij}=0 \text{ or } 1 \text{ for any } (i,j)\in A. \tag{4}$$

Let X denote the convex polyhedron defined by constraints (3) and (4).

2 Pareto optimality

We consider an order relation between vectors and define Pareto optimality based on the order relation between intervals proposed by Okada et al. [1].

Definition 1 *Let $\boldsymbol{a}$ and $\boldsymbol{b}$ be vectors on R^2 such that $\boldsymbol{a}=(a_1,a_2)$ and $\boldsymbol{b}=(b_1,b_2)$. For given α and β such that $0\le\alpha\le\beta\le1$,*

$$\boldsymbol{a}\preceq_{\alpha,\beta}\boldsymbol{b}$$

$$\iff (1-\alpha)a_1+\alpha a_2\le(1-\alpha)b_1+\alpha b_2 \tag{5}$$

$$\text{and } (1-\beta)a_1+\beta a_2\le(1-\beta)b_1+\beta b_2 \tag{6}$$

Definition 2 *The strict inequality relation in Definition 1 is defined as follows: $\boldsymbol{a}\prec_{\alpha,\beta}\boldsymbol{b}$ if $\boldsymbol{a}\preceq_{\alpha,\beta}\boldsymbol{b}$ holds and the srict inequality holds in either (5) or (6).*

The order relation $\preceq_{\alpha,\beta}$ is contented with the axiom of order relation. It is totally ordered in case of $\alpha=\beta$, while it is partially ordered in case of $\alpha\neq\beta$. Hence, this sometimes leads to the indecisive case in which neither $\boldsymbol{a}\preceq_{\alpha,\beta}\boldsymbol{b}$ nor $\boldsymbol{b}\preceq_{\alpha,\beta}\boldsymbol{a}$ holds.

Lemma 1 *If $\boldsymbol{a}\preceq_{\alpha,\beta}\boldsymbol{b}$ for $0\le\alpha\le\alpha'\le\beta'\le\beta\le1$, then $\boldsymbol{a}\preceq_{\alpha',\beta'}\boldsymbol{b}$.*

Definition 3 *Let $x,y\in X$ be two distinct feasible solutions of P1. x dominates y for given α and β iff $(c^1(x),c^2(x))\prec_{\alpha,\beta}(c^1(y),c^2(y))$ holds*

Definition 4 *Let $X_{\alpha,\beta}^d=\{x\in X\mid\exists y\in X$ such that $(c^1(y),c^2(y))\preceq_{\alpha,\beta}(c^1(x),c^2(x))$ be the set of dominated solutions of P1 for given α and β. Then $X_{\alpha,\beta}^n=X\setminus X_{\alpha,\beta}^d$ is the set of α,β-nondominated solutions of P1, or the set of α,β-Pareto Optimal solutions of P1.*

A path p_{st} corresponding to a α,β-nondominated solution x of P1 is called a

α,β-nondominated path or α,β-Pareto Optimal path. We remarks that the following equation holds :

$$\boldsymbol{d}(p_{st}) = (d^1(p_{st}), d^2(p_{st})) = (c^1(x), c^2(x))$$

where the path p_{st} corresponds to a α,β-nondominated solution x of P1.

Lemma 2 *Let $X^n_{\alpha,\beta}$ be a set of nondominated solution of P1 for given α and β. If $0 \leq \alpha \leq \alpha' \leq \beta' \leq \beta \leq 1$, then $X^n_{\alpha,\beta} \supseteq X^n_{\alpha',\beta'}$.*

3 Algorithm

On the basis of the multiple labeling method [2], an algorithm for solving bicriterion shortest path problems is immediately derived.

A label is composed of a distance vector and two pointers. Let $j \in N$ be a node of $G(N, A)$, the k-th label associated with j is $[\boldsymbol{d}_k(p_{sj}), (i, k_1)]_k$, where i $(i \neq j)$ is a predecessor node of the label, $\boldsymbol{d}_k(p_{sj})$ is the distance vector along the path p_{sj} of the k-th label of j, and k_1 indicates some label of i, for which $\boldsymbol{d}_k(p_{sj}) = \boldsymbol{d}_{k_1}(p_{si}) + \boldsymbol{c}_{ij}$.

Let P and T be sets of permanent and temporary labels respectively. An element (i, k) in the sets P or T means a pointer to the k-th label of the node i.

While the permanent labels remain unchanged, the temporary labels can be deleted during the execution of the algorithm. From a permanent label of some node $i \in N$, a temporary label is assigned to every node $j \in N$, such that (i, j) is an arc of $G(N, A)$.

Algorithm 1.

[step 0] Set the parameters α and β according to a decision maker's preference. Assign the label $[(0,0), (-,-)]_1$ to node s. Set it to temporary label and initialize the set of permanent labels to empty as follows:

$$T \leftarrow (1,1) \text{ and } P \leftarrow \emptyset.$$

[step 1] If $T = \emptyset$, go to step 3. Otherwise, among all the temporary labels determine the lexicographically smallest one. Let it be the k-th label associated with node i. Set this label as the permanent one as follows:

$$T \leftarrow T \setminus (i, k) \text{ and } P \leftarrow P \cup (i, k).$$

[step 2] While some node $j \in N$ exists, such that $(i, j) \in A$, execute

$$\boldsymbol{d}_l(p_{sj}) = \boldsymbol{d}_k(p_{si}) + \boldsymbol{c}_{ij}.$$

Let $[\boldsymbol{d}_l(p_{sj}), (i, k)]_l$ be a new temporary label of the node j. Update the temporary label as follows:

$$T \leftarrow T \cup (j, l).$$

Among all the temporary labels of node j, delete all labels representing a dominated path from s to j, and also discard the elements corresponding to the labels from the set T. Return to step 1.

[step 3] Find the nondominated paths from s to t. For that, the two pointers of each label are used to recompose backwards the list of nodes of that path until reaching the node s.

[step 4] Terminate the execution of the algorithm.

It is noted that all the α,β-nondominated paths from the source node s to all nodes $i \in N \setminus \{s\}$ can be determined with Algorithm 1.

4 Conclusion

We proposed an algorithm for solving bicriterion shortest path problem according to a decision maker's preference. Some numerical examples will be presented at the conference.

References

[1] S. Okada and M. Gen, "Fuzzy shortest path problem," *Computers and Industrial Engineering*, vol. 27, pp. 465–468, 1994.

[2] E. Martins, "On a special class of bicriterion path problems," *European Journal of Operational Research*, vol. 17, pp. 85– 94, 1984.