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BOUNDING A MULTI-SERVER QUEUE WITH A FAST SINGLE-SERVER QUEUE

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Abstract

Bounding performance measures for a multi-server queue in terms of performance measures for one or more single-server queues is a problem of long-standing interest.

In this paper, we compare a c-server queue with i.i.d. service times S_1, S_2, ..., with a corresponding fast single-server queue with the same arrival process and service times S_1/c, S_2/c, ... We call *delay* (in queue) the time between the arrival a customer and the commencement of service on that customer. Let D_{nc} be the delay of the nth arrival for either the c- or ($c = 1$)-server queue, and denote the time average delays by

$$d_c = \lim_{n \to \infty} \frac{\sum_{i=1}^{n} D_{ic}}{n}, \qquad (1)$$

when these averages exist.

We first review known *lower bounds* on various performance measures for the c-server queue in terms of corresponding quantities for the fast single-server queue. In particular, we review a sample-path bound on total work in system $V_c(t)$, which is the sum of the remaining service times of all of the customers in the system at time t. This bound will be used later in our analysis.

We then review known conditions under which the following *upper bound* holds:

$$d_c \leq d_1. \tag{2}$$

We then present new results where (2) holds for a class of $M/G/c$ queues where the service distribution has the either of the properties *Decreasing Mean Residual Life* (*DMRL*) or *New Better Than Used* (*NBU*). Service time S is *DMRL* if

$$E(S-t|S>t) \tag{3}$$

is a non-increasing function of t for $t \geq 0$, and is bounded above by $E(S)$ (so $P(S=0)=0$), and S is *NBU* if for all $t \geq 0$,

$$(S-t|S>t) \leq_{st} S. \tag{4}$$