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On the number of customers of the $Geo^{[X]}/G/1$ retrial queue with two types of customers

02302023 電気通信大学 *髙橋美佐 TAKAHASHI Misa
01006020 愛知学泉大学 大澤秀雄 ŌSAWA Hideo
01501120 電気通信大学 藤澤武久 FUJISAWA Takehisa

1. Introduction

In this note, we consider the discrete-time $Geo/G/1$ retrial queue with batch arrival and non-preemptive priorities.

Retrial queueing systems are characterized by the feature that arriving customers who find the server busy join the retrial group to try again for their requests in randam order and at random intervals. These systems are widely used in computer and communication networks. For a complete survey, see Falin [2] and Yang and Templeton[4].

Although the applications of discrete-time queues to communication and computer systems are getting importance recently, there are few study about their discrete-time systems except on T.Yang and H.Li's work [5]. So we extend their work.

2. The model

In our system, we assume that time is slotted and that all queueing activities occur at the slot boundaries. Customers depart from the system just before the slot boundaries, and arrive at the system and make retrials immediately after the slot boundaries. We consider a single server queue with batch arrivals and two types of customers, C_i $(i = 1, 2)$, which may arrive in the same batch. Furthermore we assume that C_1-customer has precedence over C_2-customer.

Let $\mathbf{B} = (B_1, B_2)$ be a random vector where B_i is the number of C_i-customers in an arriving batch. The joint distribution of the batch size vector $\mathbf{B}$, $b(j,k) = \Pr[B_1 = j, B_2 = k]$ is arbitrary where it is assumed $b(0,0) = 0$ for convenience. It is also assumed that customers arrive in a batch according to a geometric arrival process with rate λ independently of any other event in the system. That is, $\lambda = \Pr\{$ An arrival occurrs at time $m\}$. We assume further that the service time of C_i-customer follows an arbitrary distribution $\{s_i(j)\}_{j=1}^{\infty}$, *i.e.* $s_i(j) = \Pr\{$Service time of C_i-customer $= j\}$, with mean $E[S_i]$.

Under the above assumptions, we describe the behaviour of the system:

(i) If an arriving batch finds the server idle, a C_1-customer of the batch (if any) receives service and the remaining C_1-customers join a queue of infinite capacity, called priority queue, and wait to receive service in a FCFS discipline. The C_2-customers of the batch join the retrial group and repeat retrial process. The time between retrials is assumed to be distributed to independent geometric distribution with parameter $1-q$. That is $1-q = \Pr\{$ A customer makes a retrial at time $m\}$. If there are no C_1-customer in the arrival batch then a C_2-customer begins service and the remaining customers join the retrial group.

(ii) If an arrival batch finds the server busy, then the C_1- and C_2-customers of the batch join the priority queue and the retrial group respectively.

(iii) Finally, any customer in the retrial group persists to ask for service until he can recieve service.

3. Analysis

We derive the joint distribution of the numbers of customers in the priority queue and in the retrial groupe at an arbitrary time by using the supplementary variable method. To do so, let us define random variables on the system in steady state and the related probabilities.

X	:	the residual service time of the customer in service
N_1	:	the number of customers in priority queue
N_2	:	the number of customers in retrial group

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$$I \ : \ \begin{cases} \text{the type of the customer in service} \\ \qquad : \text{if there is customer in service} \\ 0 \quad : \text{if the server is idle} \end{cases}$$

$$P(i,j,k,l) = \Pr\{X^+ = i, N_1^+ = j, N_2^+ = k, I^+ = l\}$$

We introduce generating functions.

$$\begin{aligned} R_0(z) &\equiv \textstyle\sum_k P(0,0,k,0)z^k \\ Q_{j,l}(\theta,z) &\equiv \textstyle\sum_{i=1}\sum_k P(i,j,k,l)\theta^i z^k \\ B(z_1,z_2) &\equiv \textstyle\sum_k\sum_j b(j,k)z_1^j z_2^k \\ S_i(\theta) &\equiv \textstyle\sum_j s_i(j)\theta^j \\ P_l(\theta,z_1,z_2) &\equiv \textstyle\sum_{i=1} Q_{i,l}(\theta,z_2)z_1^i \qquad \text{for } l=1,2 \\ P_l'(z_1,z_2) &\equiv \tfrac{\partial}{\partial\theta}P_l(\theta,z_1,z_2)|_{\theta=0} \end{aligned}$$

By the standard procedure, we obtain the balance equations, which we omit here. And we get the set of equations as follows.

$$\begin{aligned} &\frac{\theta-(1-\lambda)-\lambda B(z_1,z_2)}{\theta}P_1(\theta,z_1,z_2) \qquad (1)\\ &= \lambda R_0(z_2)\frac{B(z_1,z_2)-B(0,z_2)}{z_1}S_1(\theta) \\ &+\frac{(1-\lambda+\lambda B(z_1,z_2))(S_1(\theta)-z_1)}{z_1}P_1'(z_1,z_2) \\ &+\frac{1-\lambda+\lambda B(z_1,z_2)}{z_1}P_2'(z_1,z_2)S_1(\theta) \\ &-\sum_{l=1}^{2}\frac{1-\lambda+\lambda B(0,z_2)}{z_1}P_l'(0,z_2)S_1(\theta) \end{aligned}$$

$$\begin{aligned} &\frac{\theta-(1-\lambda)-\lambda B(z_1,z_2)}{\theta}P_2(\theta,z_1,z_2) \qquad (2)\\ &= -(1-\lambda+\lambda B(z_1,z_2))P_2'(z_1,z_2) \\ &+\sum_{l=1}^{2}\frac{1-\lambda+\lambda B(0,z_2)}{z_2}P_l'(0,z_2)S_2(\theta) \\ &+\lambda\frac{B(0,z_2)-1}{z_2}R_0(z_2)S_2(\theta) \end{aligned}$$

$$\begin{aligned} R_0(z) &= (1-\lambda)P_1'(0,qz)+(1-\lambda)P_2'(0,qz) \\ &\quad +(1-\lambda)R_0(qz) \qquad (3) \end{aligned}$$

By substituting $\theta = 1-\lambda+\lambda B(z_1,z_2)$ and $z_1 = \varphi(z_2)$, where $\varphi(z) = S_1(1-\lambda+\lambda B(\varphi(z),z))$ into both sides in equations (1) and (2), and partial differentiating the equation (2) with respect to z_1, we obtain the z-transform of the number of customers in the system at an arbitrary time.

$$\begin{aligned} &P(1,z_1,z_2) \\ &\equiv z_1P_1(1,z_1,z_2)+z_2P_2(1,z_1,z_2)+R_0(z_2) \\ &= \frac{S_1(1-\lambda+\lambda B(z_1,z_2))(1-z_1)}{S_1(1-\lambda+\lambda B(z_1,z_2))-z_1}R_0(z_2) \\ &\cdot\frac{\lambda[B(\varphi(z_2),z_2)-B(z_1,z_2)]}{\lambda-\lambda B(z_1,z_2)} \\ &+\frac{S_2(1-\lambda+\lambda B(z_1,z_2))(1-z_2)}{S_2(1-\lambda+\lambda B(\varphi(z_2),z_2))-z_2}R_0(z_2) \\ &\cdot\frac{-\lambda(B(\varphi(z_2),z_2)-1)}{\lambda-\lambda B(z_1,z_2)} \end{aligned}$$

where

$$\begin{aligned} R_0(z) &= \prod_{n=1}^{\infty}\frac{q^n zC_2(q^n z)+1}{1-\lambda+\lambda B(0,q^n z)}(1-\lambda)R_0(0) \\ C_2(z) &= \frac{-\lambda(B(\varphi(z),z)-1)}{S_2(1-\lambda+\lambda B(\varphi(z),z))-z} \\ \varphi(z) &= S_1(1-\lambda+\lambda B(\varphi(z),z)) \end{aligned}$$

4. Concluding Remarks

We've derived the z-transform of the number of customers in the system at an arbitrary time in a $Geo^{[X]}/G/1$ retrial queue with two types of customers. The expectation of the number of customers in the system can be obtained by making use of these results.

As mentioned before, our model includes T.Yang and H.Li's model [5] as a special case where $b(l,k) = 0$ $(l \geq 1)$. And our assumption on arrival batch includes the case where C_1-customer and C_2-customer independently arrive at the system each other when the equation $b(l,k) = \Pr[B_1 = j]\Pr[B_2 = k]$ holds.

References

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