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Goal Programming Regression with Serial Correlation: Policy Implications for Japanese Telecommunications Infrastructure Development

01205520 東京理科大学 末吉 俊幸 Toshiyuki Sueyoshi 01202782 関谷 和之 Kazuyuki Sekitani*

1. Introduction

The purpose of this article is to extend their GP/CR (Goal Programming/Constrained Regression) technique further in a manner that it can incorporate Serial Correlation (SC) in GP regression. This article explores such a new research direction, because SC needs to be incorporated into a regression model in analyzing many time-series data sets. As a research extension of many conventional studies on GP regression, this article will present a new type of GP/SC model, a new algorithm and its related analytical features, along with its application to Japanese telecommunications infrastructure and its derived policy implications.

2. Goal Programming Regression with Autocorrelation

2.1. Regression and Error Structure

A regression structure and an error term incorporated in GP regression can be expressed by

$$y_t = X_t b + \epsilon_t, \qquad t = 1, \ldots, n. \tag{2.1}$$

where y_t is the observed dependent variable of the t^{th} time period, $\beta = (\beta_1, \beta_2, \ldots, \beta_m)^T$ is a column vector representing parameter coefficients to be measured, $X_t = (x_{1t}, x_{2t}, \ldots, x_{mt})^T$ is the t^{th} row vector of an observed design matrix, and η_t is an observed error in the t^{th} time period ($t = 1, \ldots, n$). This article extends the error structure in a manner that η_t is further broken down into two error components:ϵ_{t-1} and η_t. That is,

$$\epsilon_t = \rho\epsilon_{t-1} + \eta_t, \qquad t = 2, \ldots, n. \tag{2.2}$$

Here, ρ is a coefficient for a first-order SC and η_t is an unobserved, identically distributed error. The condition, $|\rho| \leq 1$, is assumed for the SC. The time period of (2.2) starts from $t = 2$ to incorporate the SC into GP formulation. Equation (2.1) can be expressed by

$$y_t = \rho y_{t-1} + [X_t - \rho X_{t-1}]\beta + \eta_t, \; t = 2, \ldots, n. \tag{2.3}$$

2.2. Goal Programming Formulation

The L_1 regression applied to (2.3) minimizes the sum of absolute error deviations for estimating ρ and β. Mathematically, it becomes

$$\begin{array}{ll} \text{minimize} & \sum_{t=2}^{n} |y_t - X_t\beta - \rho(y_{t-1} - X_{t-1}\beta)| \\ \text{subject to} & -1 \leq \rho \leq 1. \end{array} \tag{2.4}$$

Problem (2.4) can be rewritten by the following nonlinear GP problem:

$$\begin{array}{ll} \min & \sum_{t=2}^{n}[d_t^+ + d_t^-] \\ \text{s.t.} & \rho y_{t-1} + [X_t - \rho X_{t-1}]\beta + \\ & d_t^+ - d_t^- = y_t, \quad t = 2, \ldots, n \\ & -1 \leq r \leq 1 \\ & \beta: \text{ unrestricted and} \\ & d_t^+ \geq 0 \text{ and } d_t^- \geq 0, \; t = 2, \ldots, n. \end{array} \tag{2.5}$$

It is important to note that when $\rho = 0$, (2.5) becomes the original GP/CR model proposed by Charnes et al. [2]. Thus, the most important feature of (2.5) can be found in the existence of ρ in the GP formulation.

3. Computational Effort

The linear programming problem, hereafter referred to as $P(\bar{\rho})$, may be formulated as follows:

$$\begin{array}{ll} \min & \sum_{t=2}^{n}[d_t^+ + d_t^-] \\ \text{s.t.} & [X_t - \bar{\rho}X_{t-1}]\beta + d_t^+ - d_t^- = y_t - \bar{\rho}y_{t-1}, \\ & t = 2, \ldots, n, \\ & d_t^+ \geq 0 \text{ and } d_t^- \geq 0 \qquad t = 2, \ldots, n. \end{array} \tag{3.6}$$

Using (3.7), our algorithm to solve GP/SC can be summarized by the following step-by-step description:

Step 0: Obtain the lower bound (ξ_L) of GP/SC by

$$\xi_L = \min_{-1 \leq \rho \leq 1} \sqrt{\sum_{t=2}^{n}[y_t - X_t\beta - \rho(y_{t-1} - X_{t-1}\beta)]}.$$

Determine a small number (θ) and the number of intervals M.

Step 1: Set $r = 1$, $\xi_U = \infty$, $\rho_L^r = -1$, and $\rho_U^r = 1$, and $I^r = \rho_U^r = \rho_L^r$.

Step 2: Compute $\rho_i^r = \frac{\rho_U^r + \rho_L^r}{2} + \frac{I^r}{2M}i$ for all $i = -M, \ldots, M$.

Step 3: Given $\rho_i^r (i = -M, \ldots, M)$, solve problem $P(\rho_i^r)$. Let ξ_i^r be the optimal objective value of $P(\rho_i^r)$.

Step 4: Find the smallest (ξ^r) by$\xi^r = \min_{-M \leq i \leq M} \xi_i^r$. Let ρ^r and β^r be the parameter estimates attaining ξ^r.

Step 5: If $\xi^r - \xi_L \leq \theta$, then set $(\hat{\rho}, \hat{\beta}) = (\rho^r, \beta^r)$ and stop this algorithm.

Step 6: If $\xi^r - \xi_U \geq \theta$, then set $(\hat{\rho}, \hat{\beta}) = (\rho^r, \beta^r)$ and stop this algorithm. Otherwise, set $\xi_U = \xi^r$, $\rho_L^{r+1} = \max\{\rho^r - \frac{I^r}{2M}, \rho_L^r\}$, and $\rho_U^{r+1} = \min\{\rho^r + \frac{I^r}{2M}, \rho_U^r\}$.

Step 7: Set $I^{r+1} = \rho_U^{r+1} - \rho_L^{r+1}$ and $r = r + 1$. Return to Step 2.

Property 3.1. *Given any small positive number (θ), the proposed algorithm terminates itself within a finite number of iterations.*

As a methodological alternative to GP/SC, this research uses a well-known time- series technique, AR(1), "First-Order Serial Correlation," which is usually solved by the Maximum Likelihood Iterative Technique [1]. Since the AR (1) procedure is commonly incorporated into commercial softwares such as TSP (Time-Series Processor), this article omits a detailed description regarding the technique so as to avoid a descriptive duplication, except noting that its detailed description can be found in User's Guide of TSP, Version 4.2 (1994, pp. 33-35).

4. Japanese Information Infrastructure Development

4.1. Japanese Information Infrastructure

In 1990, Nippon Telegraph and Telephone (NTT) announced an ambitious future plan for connecting every home and office in Japan through a nation-wide fiber optic communications network between 1995 and 2015.

This research will focus upon the following three policy issues:

(a) Is pricing strategy the most important for the use of Japanese telecommunications network?

Table 1: Resulting Parameter Estimates

Parameter Estimates	Methodology			
	AR(1)		GP/SC	
Constant		5.592		8.017
Ave. Calling Charge	-1.138	-2.015	-1.264	-0.884
Ave. Per-Capita GNP	6.196	5.626	4.363	3.387
Access Lines	13.148	12.559	13.455	13.506
ρ	0.993	0.968	0.997	0.952
R^2	0.823	0.907	0.845	0.912

(b) Is there income elasticity in the telecommunications network?
(c) Is there externality in the telecommunications network?
hereafter, because of our data accessibility and research feasibility.

4.2. Empirical Results and Policy Implications

In an effort to empirically respond to the three policy questions, this research applies GP/SC to a real data set regarding Japanese telecommunications network. This data set conveys information concerning P (a penetration rate), W (an average calling charge), B (average per capita GNP: Growth National Product), and N (a network size). Here, the penetration rate (P) is measured by the ratio of telephone subscribers to a total population and the network size (N) is measured by the number of access lines. The average per capita GNP (B) is used to express the average Japanese spending budget. The functional form used in this empirical study can be restructured as the following linear model:

$$P_t = a + bW_t + cB_t + dN_t. \tag{4.7}$$

Here, parameter estimates to be measured for (4.7) are expressed by a, b, c and d, respectively. This empirical study incorporates the number (N) of access lines as an independent variable in (4.7) so as to express a scale effect of Japanese telecommunications network. Examining the sign and magnitude of the parameter estimate (d), this research can empirically confirm the existence and magnitude of network externality [the third policy assertion]. Furthermore, the parameter estimate "b" informs the size and magnitude of price elasticity [the second policy assertion]. Moreover, the parameter estimate "c" represents income elasticity of Japanese telephone access [the first policy assertion]. The data points used for this empirical study are sampled in the annual periods from 1953 to 1991. The sources of the data set are Miyajima [3] and NTT, Multimedia Service Promotion Headquarters. When applying GP/SC to the data set, this research needs to adjust this data set so as to avoid a case in which an independent variable with a large magnitude dominates other small ones in our computational processes. Here, the data adjustment implies that each data is divided by the data in 1973. Thus, each adjusted data is expressed in a unitless form. Resulting AR(1) and GP/SC estimates are summarized in Table 1. This table contains parameter estimates, with and without a constant parameter, for AR(1) and GP/SC. An empirical finding in Table 1 is summarized in the following manner:

Empirical Finding: [Methodology] In both the model with a constant parameter and the other model, this research cannot find any major difference between AR(1) estimates and GP/SC ones. Moreover, a high degree of R^2 scores, more than 0.823 and 0.907 in AR(1) and 0.845 and 0.912 in GP/SC, indicate the appropriateness of a model fit to the observed data set. Those empirical findings indicate a methodological validity that this research can derive policy implications from Table 1. All of them can be summarized by the following four policy implications.

Policy Implication 1 (Pricing Policy): The parameter estimate of W (Average Calling Charge) has negative signs in the four cases of Table 1, indicating that an increase in a calling charge produces a negative impact on a network penetration rate (P). Thus, pricing policy is important for the use and expansion of a telecommunications network in Japan.

Policy Implication 2 (Income Elasticity): The B (average per capita GNP) estimate becomes positive, indicating that the income of telephone subscribers has a positive impact on whether individuals want to participate into the Japanese telecommunications network. Thus, this empirical evidence confirms the existence of income elasticity.

Policy Implication 3 (Network Externality): The N (access lines) estimate becomes positive, indicating that the number of access lines has a positive impact on whether individuals want to participate into the telecommunications network. The value of telecommunications network depends upon the size of network. The more people participate into a network, the more value is added to the network. This indicates the existence of network externality.

Policy Implication 4: The magnitude of N is larger than that of the other two estimates ($W$and B). This empirical finding implies that the network externality is more important than the calling charge and the spending income in the use and growth of the Japanese telecommunications network. Many policy makers and individuals interested in Japanese information infrastructure development believe that pricing policy is the most important for the network development. However, this empirical result indicates that the size of the network is more important than the pricing policy in terms of the development of the Japanese information infrastructure. Furthermore, it is easily expected from this empirical result that the success of the future Japanese Information Highway depends upon the number of network subscribers at the initial stage of the network development.

5. Conclusion

This study has proposed a new type of GP use for time-series regression. The most important feature of the proposed GP technique is that it can incorporate SC in its time-series analysis. As an important case study, this research applies the proposed GP technique to obtain policy suggestions for the future development of Japanese information infrastructure. Empirical findings identified from the GP/SC application are summarized in four policy implications. This research believes that those policy implications may serve as a policy making basis for the future development of Japanese Information Highway.

References

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