A Composite Queue with Vacation/Set-up/Close-down Times for IP over ATM Networks

Kyoto University *SAKAI Yutaka
01304590 NTT Multimedia Networks Laboratories TAKAHASHI Yoshitaka
01304494 Nara Institute of Science and Technology TAKAHASHI Yutaka
01500044 Kyoto University HASEGAWA Toshiharu

1 Introduction

Very recently in the IP over ATM networks, we can see a complicated queueing situation where the close-down time as well as the set-up time are further needed. The close-down time here corresponds to an inactivity timer in the switched virtual channel connections (SVCC) environment. Also, the set-up time is not negligible for the ATM server. An approximate queueing result (using the M/G/1 queue) has been reported in Hassan and Atiquzzaman [3].

The main goal of this paper is 1) to present a new finite capacity queueing system with vacation, set-up, and close-down times for the SVCC operation in IP over ATM networks, generalizing the queueing system of Hassan and Atiquzzaman; 2) and to provide an exact result for the generalized system. The approach taken here is the supplementary variable technique. It should be noted that our approach enables one to treat the infinite capacity system by taking the limit of our result as the capacity (K) tends to infinity. We can derive an exact formula to the Hassan and Atiquzzaman's system [3] as a special case.

2 The model

We consider an M/G/1/K system with single vacation, set-up and close-down time, where K represents the maximum number of customers allowed in the system. The system consists of the waiting room and the server. We assume a customer arrives at the system according to a Poisson process with intensity λ independently of the system state. These arriving customers are served under the First-In-First-Out (FIFO) discipline. Since we consider the system with a finite capacity, those customers who find the system full are lost. We assume the exhaustive service discipline and the server continues to serve the customers in the queue until the system gets empty.

We call the time while the server is working a *busy period*. If the server finds no customer in the queue upon service completion, the system goes into a *close-down period*. During a close-down period, if a customer arrives, the server immediately begins service for that customer without *set-up period* and another busy period begins. On the other hand, if no customer arrives until the end of a close-down period, the system goes into a *single vacation period*. At the end of a vacation period, if the server finds any customers waiting, the server begins to serve for them after a set-up period. If the server finds no customers waiting, the system goes into an *idle period* and an arrival of a customer ends this period. Then the server begins to serve after a set-up period.

We assume service time, vacation time, set-up time, and close-down time are generally distributed as $B(x)$, $V(x)$, $S(x)$ and $C(x)$. We denote the LSTs of these distributions as $B^*(s)$, $V^*(s)$, $S^*(s)$ and $C^*(s)$, respectively. Furthermore we define $\hat{B}$ as the residual of service time and $\hat{V}$, $\hat{S}$, $\hat{C}$ are defined similarly. We denote the number of customers in the system as L and define ξ as

$$\xi \equiv \begin{cases} 1 & \text{if the system is busy} \\ 2 & \text{if in vacation} \\ 3 & \text{if doing set-up} \\ 4 & \text{if closing-down.} \end{cases}$$

Using these notations, we define

$$\begin{aligned} \Pi_{B,i}(x)dx &\equiv \mathrm{Prob}[L=i, x<\hat{B}\le x+dx, \xi=1] \\ \Pi_{V,i}(x)dx &\equiv \mathrm{Prob}[L=i, x<\hat{V}\le x+dx, \xi=2] \\ \Pi_{S,i}(x)dx &\equiv \mathrm{Prob}[L=i, x<\hat{S}\le x+dx, \xi=3] \\ \Pi_{C,i}(x)dx &\equiv \mathrm{Prob}[L=i, x<\hat{C}\le x+dx, \xi=4]. \end{aligned}$$

The LST of $\Pi_{B,i}(x)$ is expressed as $\Pi^*_{B,i}(s)$, and $\Pi^*_{V,i}(s)$, $\Pi^*_{S,i}(s)$ and $\Pi^*_{C,i}(s)$ are defined similarly.

3 Supplementary variable approach

Observing the system state at time t and $t+\Delta t$, we have the following equations.

$$\frac{d\Pi_{C,0}(x)}{dx} = \lambda\Pi_{C,0}(x) - \Pi_{B,1}(0)\frac{C(dx)}{dx}$$
$$\frac{d\Pi_{V,0}(x)}{dx} = \lambda\Pi_{V,0}(x) - \Pi_{C,0}(0)\frac{V(dx)}{dx}$$
$$\frac{d\Pi_{V,i}(x)}{dx} = \lambda\Pi_{V,i}(x) - \lambda\Pi_{V,i-1}(x)$$
$$\frac{d\Pi_{V,K}(x)}{dx} = -\lambda\Pi_{V,K-1}(x)$$
$$\frac{d\Pi_{S,1}(x)}{dx} = \lambda\Pi_{S,1}(x) - \left(\lambda\Pi_0^* + \Pi_{V,1}(0)\right)\frac{S(dx)}{dx}$$
$$\frac{d\Pi_{S,i}(x)}{dx} = \lambda\Pi_{S,i}(x) - \lambda\Pi_{S,i-1}(x) - \Pi_{V,i}(0)\frac{S(dx)}{dx}$$
$$\frac{d\Pi_{S,K}(x)}{dx} = -\lambda\Pi_{S,K-1}(x)$$
$$\frac{d\Pi_{B,1}(x)}{dx} = \lambda\Pi_{B,1}(x) - \Big(\Pi_{B,2}(0) + \Pi_{S,1}(0) + \Pi_{C,1}(0)\Big)\frac{B(dx)}{dx}$$
$$\frac{d\Pi_{B,i}(x)}{dx} = \lambda\Pi_{B,i}(x) - \lambda\Pi_{B,i-1}(x) - \left(\Pi_{B,i+1}(0) + \Pi_{S,i}(0)\right)\frac{B(dx)}{dx}$$
$$\frac{d\Pi_{B,K}(x)}{dx} = -\lambda\Pi_{B,K-1}(x) - \Pi_{S,K}(0)\frac{B(dx)}{dx}$$

where Π_0^* is the probability that the system is idle at an arbitrary time. Equating the rate at which a vacation period ends with no customers present to the rate at which the idle period following it ends because of arrivals, we have

$$\lambda\Pi_0^* = \Pi_{V,0}(0). \tag{1}$$

During the close-down time, an arriving customer may end the period, thus

$$\Pi_{C,1}(0) = \lambda\int_0^\infty \Pi_{C,0}(x)dx. \tag{2}$$

(1) and (2) yield the following LSTs.

$$(\lambda - s)\Pi_{C,0}^*(s) = -\Pi_{C,0}(0) + \Pi_{B,1}(0)C^*(s)$$
$$(\lambda - s)\Pi_{V,0}^*(s) = -\Pi_{V,0}(0) + \Pi_{C,0}(0)V^*(s)$$
$$(\lambda - s)\Pi_{V,i}^*(s) = -\Pi_{V,i}(0) + \lambda\Pi_{V,i-1}^*(s)$$
$$-s\Pi_{V,K}^*(s) = -\Pi_{V,K}(0) + \lambda\Pi_{V,K-1}^*(s)$$
$$(\lambda - s)\Pi_{S,1}^*(s) = -\Pi_{S,1}(0) + \left(\Pi_{V,0}(0) + \Pi_{V,1}(0)\right)S^*(s)$$
$$(\lambda - s)\Pi_{S,i}^*(s) = -\Pi_{S,i}(0) + \lambda\Pi_{S,i-1}^*(s) + \Pi_{V,i}(0)S^*(s)$$
$$-s\Pi_{S,K}^*(s) = -\Pi_{S,K}(0) + \lambda\Pi_{S,K-1}^*(s) + \Pi_{V,K}(0)S^*(s)$$

$$(\lambda - s)\Pi_{B,1}^*(s) = -\Pi_{B,1}(0) \Big(\Pi_{B,2}(0) + \Pi_{S,1}(0) + \lambda\Pi_{C,0}^*(0)\Big)B^*(s)$$

$$(\lambda - s)\Pi_{B,i}^*(s) = -\Pi_{B,i}(0) + \lambda\Pi_{B,i-1}^*(s) + \left(\Pi_{B,i+1}(0) + \Pi_{S,i}(0)\right)B^*(s)$$

$$-s\Pi_{B,K}^*(s) = -\Pi_{B,K}(0) + \lambda\Pi_{B,K-1}^*(s) + \Pi_{S,K}(0)B^*(s).$$

4 Performance measures

We obtain the system state probability at an arbitrary point, $\Pi_{C,0}^*(0)$, $\Pi_{V,i}^*(0)(0 \le i \le K)$, $\Pi_{S,i}^*(0)(1 \le i \le K)$, and $\Pi_{B,i}^*(0)(1 \le i \le K)$ after algebraic manipulation of the above equations. We also obtain $\Pi_{C,0}(0)$, $\Pi_{V,i}(0)(0 \le i \le K)$, $\Pi_{S,i}(0)(1 \le i \le K)$ and $\Pi_{B,i}(0)(1 \le i \le K)$ as by-products. Then three important performance measures of the system, the loss probability of an arbitrary customer P_{loss}, the set-up rate γ_{set-up}, which is the number of set-up periods per unit time, and the LST of waiting time distribution of a customer accepted by the system $W(s)$ are given as

$$P_{loss} = \Pi_{S,K}^*(0) + \Pi_{V,K}^*(0) + \Pi_{B,K}^*(0)$$

$$\gamma_{set-up} = \frac{\sum_{i=1}^K \Pi_{S,i}^*(0)}{-S^{*(1)}(0)}$$

$$W^*(s) = \Bigg(\Pi_{C,0}^*(0) + \Pi_0^* S^*(s) + \sum_{i=0}^{K-1}\Pi_{V,i}^*(s)[S^*(s)][B^*(s)]^i + \sum_{i=1}^{K-1}\Pi_{S,i}^*(s)[B^*(s)]^i + \sum_{i=1}^{K-1}\Pi_{B,i}^*(s)[B^*(s)]^{i-1}\Bigg)/(1 - P_{loss}).$$

References

[1] B. Doshi, *Stochastic Analysis of Computer Communication Systems*, H. Takagi (editor), 217-265, Elsevier Science Publishers B.V., Amsterdam (1990).

[2] A. Frey and Y. Takahashi, *Opns Res. Letters*, **16** (to appear).

[3] M. Hassan and M. Atiquzzaman, *IEICE Trans. Commun.*, **E80-B**, 317-323 (1997).

[4] T.T. Lee, *Opns Res.*, **32**, 774-784 (1984).

[5] M. Miyazawa, *Stochastic Models*, **10**, 389-413 (1994).

[6] H. Takagi, *Queueing Analysis, A Foundation of Performance Evaluation, Vol.1: Vacation and Priority Systems*. Elsevier Science Publishers B.V., Amsterdam (1991).