An alternative method for Constrained Facet Analysis (CFA)on DEA

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1. Introduction

Data Envelopment Analysis (DEA) proposed by-Charnes et al. in 1978 [1], is an evaluating method for systems having multiple inputs and outputs.

DEA is said to be a data oriented analytical method, it constructs the efficient frontier for existing coordinates. In the region where no efficient DMU exist, a frontier cannot be constructed, but spreading some slack hyperplanes, we can obtain an improvement point with slack value for some inefficient DMUs on the efficient frontier. This means DEA only considers improvement points on already existing data region.

To solve this problem, Bessent et al. have proposed the concept of Constrained Facet Analysis (CFA) [2]. CFA can extend the frontier of DEA, to region where no data exists. Using this concept, we can analyze more widely. But their algorithm is complex and sometimes we cannot get correct solutions.

In this paper, we propose an alternative way to obtain a frontier based on CFA concept.

2. Basic concepts

2.1. DEA

DEA (output-oriented BCC model) may be formulated as follows:

[P1]

$$\min \quad \eta_a - \varepsilon\left(\sum_{i=1}^{m} s_{ia} + \sum_{r=1}^{k} s_{ra}\right)$$

$$\text{s.t.} \quad X_{ia} = \sum_{j=1}^{n} \lambda_{ja} X_{ij} + s_{ia} \quad (i = 1,..,m)$$

$$\eta_a Y_{ra} = \sum_{j=1}^{n} \lambda_{ja} Y_{ij} - s_{ra} \quad (r = 1,..,s)$$

$$\sum_{j=1}^{n} \lambda_{ja} = 1$$

$$\lambda_{ja} \geq 0 \qquad (j = 1,..,n)$$

where X_{ij} is DMU$_j$'s ith input value, Y_{rj} is DMU$_j$'s rth output value, ε is a non archimedean number, s_{ia} and s_{ra} are slack variables of DMU a's input i and output r, respectively, and λ is a non negative connective variable.

In this model, each DMU is judged, to be efficient if η_a equals 1, and inefficient, η_a is less than 1. η_j represents the across-the-board expanding ratio for efficient frontier. The optimal objective function value η_a^* is the inverse number of the efficient value, so the efficient value of DMU$_j$ is $1/\eta_j$ *.

There are some cases that an improvement point cannot be obtained from the efficient frontier only any across-the-board expanding for. In this case, it is possible to reach the efficient frontier by given some positive value to some slack.

2.2. CFA

Constrained Facet Analysis concept has been proposed by Bessent et al.

For example, in Fig.1, the original frontier is composed from the segment AB and BC, and there is a parallel line from point C for slack hyperplane.

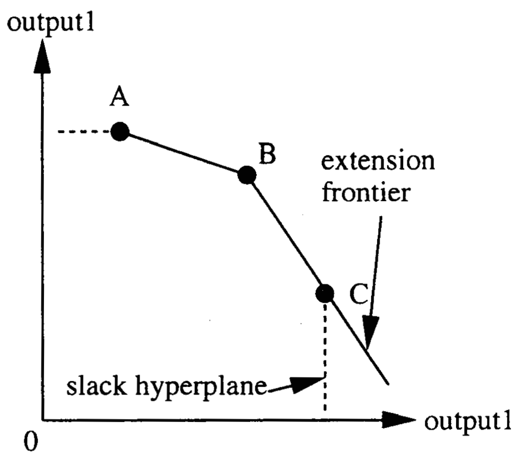

Fig.1.

The CFA concept is to obtain the line extended segment BC. Then we can find some improvemnent point on the extended line.

When the DEA structure is of low dimension, as in Fig.1., calculation is easy. But when the dimension is high, the structureis sometimes too complex for analysis.

There are some faults of the original CFA. First, it can only be applied the CCR model. Second, analysis is impossible, if there are nondiscretionary items. Third, sometimes we cannot obtain an improvement point using the input–oriented model.

3. Our method

In this section, we describe our new method to obtain an expanded frontier based on CFA concept.

Our purpose is to obtain new expanded frontiers for region having slack hyperplane, while keeping original efficient frontiers. Moreover, these frontiers are found as extending hyperplane which construct the efficient frontier.

So, our method is to find which hyperplane of the efficient frontier should be extended for it. To find this answer, we consider the following problem:

[P2]

$$\min \quad \theta_a$$

$$\text{s.t.} \quad \sum_{j \in E} X_{ij}(\lambda_{kj}^{+} - \lambda_{kj}^{-}) = X_{ik} \quad (k \in \{E, a\})$$

$$\sum_{j \in E} Y_{rj}(\lambda_{kj}^{+} - \lambda_{kj}^{-}) = \theta_k Y_{rk} (k \in \{E, a\})$$

$$\sum_{j \in E} (\lambda_{kj}^{+} - \lambda_{kj}^{-}) = 1 \qquad (k \in \{E, a\})$$

$$\theta_k \geq 1 (k \in \{E, a\})$$

$$\lambda_{kj}^{+} + \lambda_{kj}^{-} - M z_j \leq 0 \qquad (k \in \{E, a\})$$

$$\sum_{j \in E} z_j = m + s \quad (j \in E \ ; \ k \in \{E, a\})$$

$$z_j \in \{0, 1\} \qquad (j \in E)$$

$$\lambda_{kj}^{+}, \lambda_{kj}^{-} \geq 0$$

where M is a very large number, m and s are the numbers of input and output, respectively, and E is the set of subscrispts such of the efficient DMUs.

In this model, if z_j equals 1, then DMU_j is an element which defines the hyperplane for the improvement point of DMU_a. The number of elements of the reference set of DMU_a equals the sum of the number of input items and output ones. For this formula, the hyperplane defining the improvement point concerned with inefficient DMU can be fixed uniquely, without degeneracy.

Further, $(\lambda_{kj}^{+} - \lambda_{kj}^{-})$ can take values not only nonnegative value but also negative. It means it permits externally dividing point. So, an improvement point on the extended boundary of the efficient frontier. If $(\lambda_{kj}^{+} + \lambda_{kj}^{-})$ is not zero (larger than zero) then z_j equals 1. Conversely, if $(\lambda_{kj}^{+} + \lambda_{kj}^{-})$ is zero then z_j equals zero.

4. Conclusion

In this paper, we proposed an alternative (and new) method to obtain an extended frontier based on CFA concept.

But the extended frontier passes through negative area, as before. There remaineds the problem that there is some possibility that no inprovement point can be obtained in the input–oriented case.

To solve this problem, by combining multiplicative model and CFA concept, we limit efficient frontier within positive area [3].

References

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