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SINGLE-LEVEL STRATEGIES FOR FULL-INFORMATION BEST-CHOICE PROBLEMS. II

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ABSTRACT. Continuing the work in the previous paper, Part I, we discuss some full-information best-choice problems and their extension to two-player competitive situation. Three problems are formulated and solved. (1) Best-choice problem where the objective is to select the k bests among sequentially arriving n *r.v.s*, *iid* with common uniform distribution on $[0,1]$, (2) zero-sum best-choice game where each player's objective is to select a *r.v.* larger than one chosen by the opponent and players' priority is given in advance, and (3) non-zero-sum game variant of (2), where each player is given his own sequence of *r.v.s* individually. The strategies allowed are restricted to the single-level strategies, and in (1) and (2) the total number, N, of sequentially arriving *r.v.s* is also a *r.v.* obeying geometric distribution with parameter θ.

§1 Introduction

§2 Choosing one of the k bests

§3 Zero-sum best-choice game.

Better-than-opp./Players-priority/Common/ZS, with FI.

Players observe a common seq. $\{X_t\}_1^N$. Players I(II) chooses $z(w) \in [0,1]$, and accepts the earliest r.v. $\in (z,1]$ ($(w,1]$).

If the earliest r.v. $\in (z \vee w, 1]$ appears, it is acc. by I, and I drops out thereafter. A player stopping at the larger r.v. than the opp. is the winner, and gets 1 from the opp. I(II) wants to maxi(mini)-mize I's exp. payoff. We find that

$$M_n(z,w) = \begin{cases} (w-z)\sum_{i=1}^{n} z^{i-1}(2w^{n-i}-1) + (1-w)\sum_{i=1}^{n} z^{i-1}w^{n-i} & \text{if } z<w \\ (z-w)\sum_{i=1}^{n} w^{i-1}(1-2z^{n-i}) + (1-z)\,n\,w^{n-1} & \\ \quad + \dfrac{(1-z)(z-w)}{1-w}\sum_{i=1}^{n-1} w^{i-1}(1-w^{n-i}) & \text{if } z>w, \end{cases}$$

After taking $E_N M_N(z,w)$ and transformations to $(s,t) \in [\theta,1]^2$, we get

$$(1-\theta)\,M(s,t) = \begin{cases} (t^{-1}-1-t)s + 2t - 1, & \text{if } \theta<s<t \\ 1-2s+t-(s^{-1}-1)^2 t^2, & \text{if } \theta<t<s \end{cases}$$

is a cont. game on the square $[\theta,1]^2$, with $M(s,s) = (1-\theta)^{-1}s(1-s)$ on the diagonal.

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Th. 2 The game has a pure-str. solution (s_0, t_0) with

$$s_0 = 1 - 1/\sqrt{5} \fallingdotseq 0.55278, \quad t_0 = (\sqrt{5}-1)/2 \fallingdotseq 0.61803,$$

i.e. $z^* = [1 - 0.809040\theta(1-\theta)^{-1}]^+$, $w^* = [1 - 0.61803\,\theta(1-\theta)^{-1}]^+$

The value of the game is

$$M(s_0, t_0) = (\sqrt{5}-2)(1-\theta)^{-1} \fallingdotseq 0.23607(1-\theta)^{-1}.$$

§4 Non-zero-sum best-choice game

Better-than-opp. / Each / NZS, with FI.

I(II) observes his own $\{X_t\}$ ($\{Y_t\}$). A player stopping at the larger r.v. than the opponent is the winner, and gets 1 from the opponent. I(II) wants to maximize his prob. of win. We find that

$$(*) \quad M_1(z, w) = \begin{cases} (1-z^n)\left\{\dfrac{(1-w)(1-w^n)}{2(1-z)} + w^n\right\}, & \text{if } z < w \\ (1-z^n)\left\{\dfrac{(1-2w+z)(1-w^n)}{2(1-w)} + w^n\right\}, & \text{if } z > w \end{cases}$$

$$M_2(z, w) = M_1(w, z).$$

i.e. a continuous game on the unit square $[0,1]^2$, with $M_1(z,z) = \frac{1}{2}(1-z^{2n})$, on the diagonal.

Th. 3 The game has an eq. pt. (u_0^{-1}, u_0^{-1}), and common eq. value $\frac{1}{2}(1-u_0^{-2n})$, where u_0 is a unique root in $(1,\infty)$ of the equation

$$(u^n-1)^2 = n(u-1)(u^n+1), \quad \text{i.e.} \quad \frac{1}{n}\sum_{i=0}^{n-1} u^i = \frac{u^n+1}{u^n-1}$$

Therefore $P_r(\text{draw}) = u_0^{-2n}$

$n=10$	$u_0^{-1} = 0.8955$;	$\frac{1}{2}(1-u_0^{-2n}) = 0.4559$,	$u_0^{-2n} = 0.0873$
20	0.9420	0.4541	0.0918
50	0.9767	0.4528	0.0945

Two objectives selecting-best and better-than-opp. do not go along with other. Another way to confirm this is: Let us take $z = e^{-a/n}$, $w = e^{-b/n}$ with $a, b > 0$ in (*). Then

$$M_1(a,b) = \begin{cases} (1-e^{-a})\left\{\dfrac{b}{2a}(1-e^{-b}) + e^{-b}\right\}, & \text{if } 0 < b < a \\ (1-e^{-a})\left\{1 - \dfrac{a}{2b}(1-e^{-b})\right\}, & \text{if } 0 < a < b \end{cases}$$

i.e. a cont. game on $(0,\infty)^2$, with $M_1(a,a) = \frac{1}{2}(1-e^{-2a})$.

Th. 4 The game has an eq. pt. (a_0^{-1}, a_0^{-1}), with $a_0^{-1} \fallingdotseq 0.8544$, and common eq. value $\frac{1}{2}(1-e^{-2/a_0}) \fallingdotseq 0.4095$. Therefore $P_r(\text{draw}) = e^{-2/a_0} \fallingdotseq 0.1811$, where $a_0 \fallingdotseq 1.1704$ is a unique root in $(1,\infty)$ of the equation

$$2(a-1)e^{-a} = \sqrt{a^2+8a} - a - 2.$$

（以下略）

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