Markov Decision Process Models in Medical Decision Making

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1. Introduction

In 1983, Beck and Pauker [1] introduced the use of discrete-time Markov models for determining medical prognosis and, since then, such Markov models become a major tool in medical decision analysis. Markov models are particularly useful when a decision involves risk that is ongoing over time, when the times at which important events will occur are uncertain, and when those events may happen repeatedly. A major deficiency of simple Markov models is, however, that they cannot lead to decision making directly.

To overcome this, Markov decision process (MDP) models will be useful and it is the aim of this presentation to describe the use of MDP models for determining optimal medical treatment processes. The model is applied to arteriosclerosis obliterans to demonstrate its usefulness where there are a few possible treatments in each Markov state of health.

2. The MDP Model

The theory of MDP's is well established and has been applied to medical problems [2, 3]. The MDP model provides an optimal medical treatment at each state and at each time epoch. The model assumes that an *individual* patient is always in one of Markov states of health, and all events of interest are modelled as transitions from one state to another.

The general problem considered here is the history of a chronic disease and a medical treatment for it, which can be viewed as a sequence of particular states of health and actions of decision. Suppose that all distinct states of health are enumerated as $\{1, 2, \cdots, K\}$, where K signifies the absorbing state, meaning *death*. In each state s, there are available a set of possible (finite) medical treatments (including no surgical treatment), called *actions*, and they are enumerated as $\{1, 2, \cdots, L_s\}$.

In the MDP setting, the transition probability $p_{sj}^a(t)$ is associated with not only states but also action a. It is important to consider time-dependent transition probabilities to incorporate the factor of increasing age, since the mortality of the healthy population increases exponentially with age [4]. Note that implicit in $p_{sj}^a(t)$ is that the process has no memory of *prior actions* as well as prior states. This 'Markov property' might be controversial and should be investigated before applied.

The reward (or utility) is a function of time t, state s and action chosen a: $r_t(s, a)$. It is assumed that $r_t(K, a) = 0$ and rewards are discounted to represent the fact that later events have less impact than earlier ones. The objective is to determine a policy so as to maximize the expected total discounted rewards:

$$\max E\left[\sum_{t=0}^{N-1} \rho^t\, r_t(X_t, A_t) + \rho^N\, r_N(X_N)\right] \quad (1)$$

where N is the terminal time epoch and $r_N(s)$ is the terminal reward. What is nice to consider (1) is that there is available a simple but efficient algorithm to solve it, called the *backward induction algorithm*. Note that (1) is not quite accurate because the absorption time may occur after N.

However, since the mortality increases exponentially with age, (1) should be a good approximation if N is sufficiently large.

3. A Detailed Example

Arteriosclerosis Obliterans (ASO) are a chronic disease and, according to the Duropean Consensus Document, a patient is classified into either intermittent claudication (IC) or critical leg ischaemia (CLI), and the condition of the disease gets worse in this order. If the condition becomes much worse, the patient ought to undergo an amputation (AMP). This disease coexists with cerebral arterial diseases or cardiovascular diseases, which sometimes cause death [5].

Physical exercise or pharmacological treatment of ASO may be primary. Concomitant to surgical therapy are either revascularisation or percutaneous catheter procedures. According to a medical doctor (private communication), however, repeated attempts to unblock the graft should be avoided in each condition. An AMP should be undertaken if such surgical therapies have failed.

3.1. Markov States

Taking the characteristics of the disease mentioned above, we define the Markov states, which will be shown during the presentation.

3.2. Actions

The second step to construct the MDP model is to enumerate all possible actions in each state. For example, in the IC state, surgical therapies are possible and we consider three actions in this analysis; percutaneous transluminal angioplasty (PTA), bypass surgery (BP), or no surgical treatment (NT). In the well states, on the other hand, the NT action other than pharmacotherapy is only possible. In this way, we recognize possible actions in every state and the details will be shown in the presentation.

3.3. Transition Probabilities

The transition probabilities used in this analysis are abstracted from the clinical literature such as [6, 5]. The cycle length chosen for the MDP model is one year. We employ the Gompertz function to handle changing mortality probabilities [4]:

$$f(t) = 1 - \exp\{-(m + ke^{rt})\}$$

where m is a disease specific mortality rate, and k and r are the coefficients depending on the race and sex. Our MDP model is non-homogeneous only through this mortality probability. This is because we could not find data for time-dependent transition probabilities in the literature but, if any, it is a simple matter to incorporate such factors into our MDP model. The details will be given in the presentation.

3.4. Calculation of QALY's

For a given policy of actions at all the states, we can evaluate quality-adjusted life years (QALY's) by the standard cohort simulation method. The utilities corresponding to health conditions used in our analysis are obtained from [7]. Numerical results will be reported in the presentation.

References

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