1998年度日本オペレーションズ・リサーチ学会
秋季研究発表会

迷惑施設のパレート最適集合

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1. INTRODUCTION

We are concerned with the bicriteria location model with minimax and maximin criteria. The aim of this paper is to characterize the efficient sets associated the bicriteria model, and to present a analytical procedure for generating these efficient sets. It will be shown that the efficient set can be analytically expressed with the help of the nearest-point and farthest-point Voronoi diagrams.

2. LOCATION MODELS

Let m population centers $\mathbf{p}_1, \cdots, \mathbf{p}_m$ be located on the plane. In the *maximin* location problem, a facility is established within feasible region Ω so as to maximize the Euclidean distance from the facility to its nearest center as follows:

$$\max_{\mathbf{x}\in\Omega} F(\mathbf{x}) = \min_{i\in\{1,\cdots,m\}} \|\mathbf{x}-\mathbf{p}_i\|. \tag{1}$$

This minimizer, denoted by $\mathbf{a}^*$, is called a *anti-center*. Let n demand points $\mathbf{q}_1, \cdots, \mathbf{q}_n$ be located in the plane. In the *minimax* location problem, a facility is set up in order to minimize the farthest Euclidean distances from the facility to these demand points. Mathematically, this is

$$\min_{\mathbf{x}} G(\mathbf{x}) = \max_{i\in\{1,\cdots,n\}} \|\mathbf{x}-\mathbf{q}_i\|. \tag{2}$$

Note that the minimizer of $G(\mathbf{x})$, denoted by $\mathbf{c}^*$ exists uniquely.

3. EFFICIENT SET

The steps in this method are given below:

1. Construct the nearest-point Voronoi diagram of $\mathbf{p}_1, \cdots, \mathbf{p}_m$, and the farthest-point Voronoi diagram of $\mathbf{q}_1, \cdots, \mathbf{q}_n$.
2. Set E={$e|e$ is nearest-point Voronoi edge} $\cup$ {$e|e$ is farthest-point Voronoi edge}.
3. In the criterion space, find the lower envelope of the curves corresponding to the edges within E, and replace E by the set of edges or subedges corresponding to the curves generating this lower envelope.

These situations are illustrated in Figures 1 and 2. The total time complexity is $O(mn \log mn)$. Two remarks are in order. First, the tradeoff curve consists of piecewise quadratic curves. In particular, the tradeoff curve corresponding to the nearest-point Voronoi edge becomes convex. Second, jumps of the tradeoff curve occur only at the point corresponding to nearest-point Voronoi vertex. Therefore, in the scalarized location problem in which the objective is to minimize a convex combination of the maximin and minimax objective functions with suitable weights assigned to each of the two objectives, Voronoi vertices within S^* tend to be optimal for this scalarized problem.

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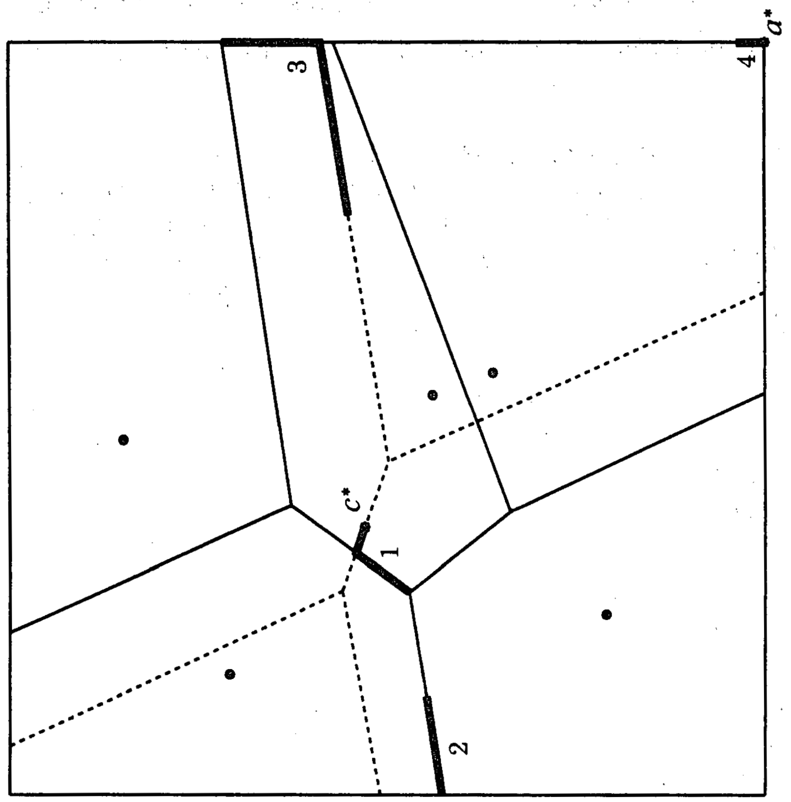

Figure 1: Voronoi diagrms

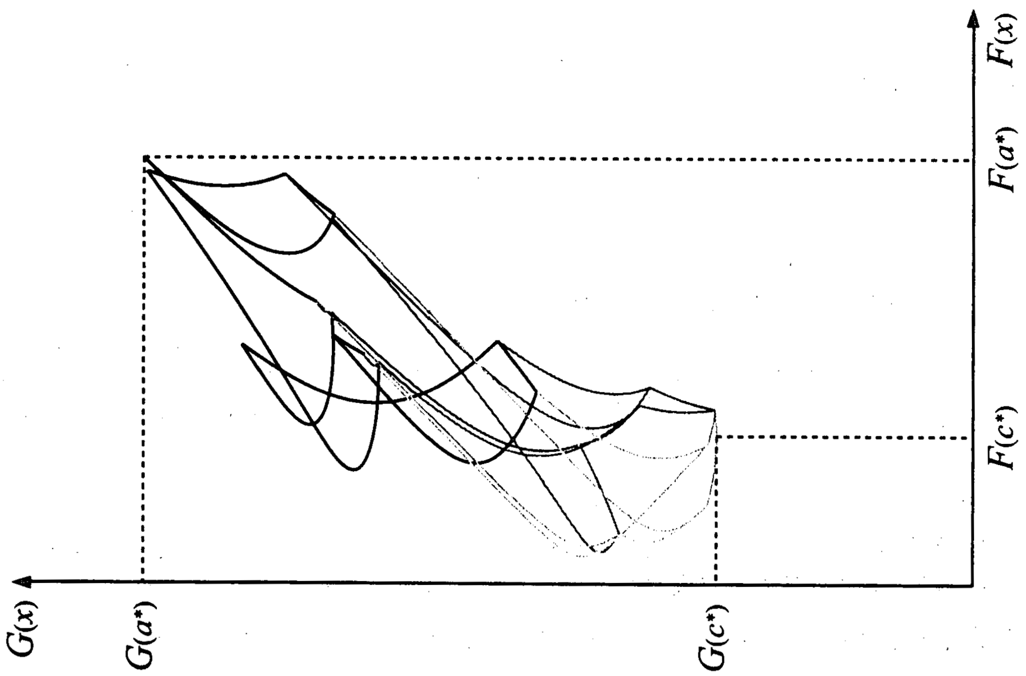

Figure 2: Criterion Space