Multiple Choice Problems Related to the Duration of the Secretary Problem

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1. Multiple choice duration problem

Though Ferguson, Hardwick and Tamaki[1] considered the various duration models extensively, they confined themselves to the study of the one choice problems. In this paper, we attempt to extend the one choice problems to the multiple choice problems. For the m choice duration problem, we are allowed to choose at most m objects sequentially, and receive each time a unit payoff as long as either of the chosen objects remains a *candidate*(for simplicity we refer to a relatively best object as a candidate). Obviously only candidates can be chosen, the objective being to maximize the expected payoff.

It can be shown that, for the m choice duration problem, there exists a sequence of integer-valued critical numbers $(s_1, s_2, \ldots, s_m)$ such that, whenever there remain k choices yet to be made, then the optimal strategy immediately selects a candidate if it appears after or on time s_k, $1 \le k \le m$. It is also shown that s_k is non-increasing in k. s_k/n converges to some definite value s_k^* and a recursive formula for calculating s_k^* in terms of $s_1^*, s_2^*, \ldots, s_{k-1}^*$ will be given by

$$s_k^* = \exp\left[-\left\{1+\sqrt{1-2\sum_{i=1}^{k-1}\frac{[(k-i+2)+(k-i+1)\log s_i^*]}{(k-i+2)!}(\log s_i^*)^{k-i+1}}\right\}\right].$$

Table 1. The asymptotic critical number s_m^* for some values of m and c

c	β	s_1^*	s_2^*	s_3^*	s_5^*	s_{10}^*	$s_\infty^*(=\beta')$
0.0	1.0000	0.1353	0.0799	0.0493	0.0199	0.0024	0.0000
0.1	0.8942	0.1513	0.0990	0.0698	0.0416	0.0281	0.0280
0.2	0.7717	0.1754	0.1294	0.1047	0.0839	0.0787	0.0787
0.3	0.6130	0.2208	0.1898	0.1761	0.1690	0.1684	0.1684

2. Multiple choice duration problem with an acquisition cost

The multiple choice duration problem is generalized by imposing a constant acquisition cost c(>0) each time an object is chosen. The optimal strategy is similar to that of the problem considered in Section 1. Let β be the unique root $x \in [e^{-1}, 1)$ of the equation $-x\log x = c$ for $c \le e^{-1}$. Then s_k^* satisfies the following recursive relation

$$s_k^* = \exp\left[-\left\{1+\sqrt{(1+\log\beta)^2-2\sum_{i=1}^{k-1}\frac{[(k-i+2)B_{k+1,i}+(k-i+1)B_{k+2,i}]}{(k-i+2)!}}\right\}\right],$$

where $B_{k,i} = (\log s_i^*)^{k-i} - (\log\beta)^{k-i}$. Let q_m^*, $m \ge 1$, be the expected net payoff for the m choice duration problem when n tends to infinity. Then we have

$$q_m^* = -\left(\sum_{k=1}^{m} s_k^* \log s_k^* + mc\right).$$

Table 2. The asymptotic expected net payoff q_m^* for some values of m and c

c	q_1^*	q_2^*	q_3^*	q_5^*	q_{10}^*	q_∞^*
0.0	0.2707	0.4725	0.6208	0.8066	0.9656	1.0000
0.1	0.1858	0.3147	0.4005	0.4871	0.5195	0.5197
0.2	0.1053	0.1700	0.2062	0.2322	0.2363	0.2363
0.3	0.0335	0.0489	0.0547	0.0569	0.0570	0.0570

3. Multiple choice duration problem with a replacement cost

In this section, we are allowed to possess only one object at a time and a constant cost d(>0) is incurred each time replacement takes place. It can be shown that, for the m choice problem, there exists a sequence of integer-valued critical numbers $(s_1,s_2,\ldots,s_{m-1},t_m)$ such that the optimal strategy first selects a candidate that appears after or on time t_m and then it replaces the previously chosen object with a new candidate that appears after or on time s_k, but no later than c(n) if k more replacements are availabke. s_k/n and t_m/n respectively converges to some definite values s_k^* and t_m^* and they are given as follows if δ is defined as the unique root $x\in[e^{-1}, 1)$ of the equation $-x\log x=d$ for $d\le e^{-1}$.

$$s_k^* = \exp\left[-\left\{1+\sqrt{(1+\log\delta)^2 - 2\sum_{i=1}^{k-1}\frac{[(k-i+2)B_{k+1,i}+(k-i+1)B_{k+2,i}]}{(k-i+2)!}}\right\}\right]$$

$$t_m^* = \exp\left[-\left\{1+\sqrt{(1+\log s_m^*)^2-(2+\log\delta)\log\delta}\right\}\right]$$

where $B_{k,i} = (\log s_i^*)^{k-i} - (\log\delta)^{k-i}$.

Table 3. The asymptotic critical number t_m^* for some values of m and d

d	t_2^*	t_3^*	t_5^*	t_{10}^*	t_∞^*
0.1	0.0916	0.0656	0.0397	0.0270	0.0268
0.2	0.1063	0.0885	0.0725	0.0684	0.0684
0.3	0.1243	0.1186	0.1154	0.1151	0.1151

Table 4. The asymptotic expected net payoff r_m^* for some values of m and d

d	r_2^*	r_3^*	r_5^*	r_{10}^*	r_∞^*
0.1	0.4047	0.4934	0.5828	0.6166	0.6168
0.2	0.3435	0.3845	0.4146	0.4198	0.4198
0.3	0.2927	0.3017	0.3056	0.3059	0.3059

Reference [1] Ferguson, T.S., Hardwick, J.P. and Tamaki, M.(1992) Maximizing the duration of owning a relative best object. Contemporary Mathematics 125, 37-57.