1998年度日本オペレーションズ・リサーチ学会
秋季研究発表会

Multiple Risk Assessment with Random Utility Models in Group Decision Making

01212894 Department of Business Administration & Informatics, Setsunan University *SEO Fumiko

1. Introduction

This paper discusses a probabilistic utility approach for multiple risk assessment in group decision making.

In the group decision making, the difficulty of the construction of group utility functions based on individual preference orders has been raised. The probabilistic utility approach based on the random utility models concerns the randomness of the subject' response to the presentation of the choice problems. Individual decision makers in group decisions are presumed to be all anonymous and to possess their unknown and diversified preference structures. As a result, the choice behavior by the individuals are treated as uncertain events. The probabilistic group utility models are defined with the probability distributions called the preference probability.

The multiple risk assessment for alternative gamble prospects in the incomplete information structure is discussed in terms of the multiple risk evaluation function via the probabilistic value tradeoffs.

2. Probabilistic Choice Behavior and Random Utility Models

2.1 Probabilistic Choice Behavior

Let Ψ denote the universal set of possible information and ϖ, $\varpi \subseteq \Psi$, be its obtainable subset which defines an information structure. Denote by $\eta \in \varpi$ its element. Let A denote the universe of choice behavior, or actions, and F, $F \subseteq A$, be its subset recognized as the feasible. Let X denote the universe of objects for choice and ξ, $\xi \subseteq X$, be its known, or discriminating, subset. Denote by $a \in A$ and $x \in X$ their elements respectively. Note that A is a set of decision alternatives and X is a set of the multiple attributes regarded as the certain variables. When the subject (IDM) is presented an information as the stimulus, the choice behavior of the subject is the probabilistic response to it among the decision alternatives. The primal choice model is defined with the attribute function $x(a) = x(a \mid \eta)$ on the feasible decision alternative $a \in F$ when an obtainable information structure $\varpi \ni \eta$ is given. The information structure ϖ is enlarged and revised. Subsequently the same for the sets of the feasible actions $F(\varpi)$, and the discriminating outcomes ξ (ϖ). In this paper as the first discussion, however, the changes of the information structure are not treated.

In group decision making, the collective response of the subjects is assumed to be unknown and revealed by probabilistic mechanism. In probabilistic choice model, the response probability is defined as the *choice probability* which naturally obeys to the general probability rule.

$$p_F(a|\eta) \ge 0,$$
$$\sum_{a\in F} p_F(a|\eta) = 1 \text{ for all } a \in F \subseteq A \tag{1}$$

where $p_F(a)$ denotes a probability of an action a to be chosen from the feasible set $F \subseteq A$. From our primary choice model, the choice probability is written equivalently as

$$p_\xi(x(a|\eta)) \ge 0,$$
$$\sum_{x\in\xi} p_\xi(x(a|\eta)) = 1 \text{ for all } x \in \xi \subseteq X. \tag{2}$$

Hereafter, the a and η are omitted in the outcome function $x(a \mid \eta)$. Then $p_\xi(x)$ is used as the choice probability, which represents a probability for an outcome x to be chosen in ξ. In the case of the binary preference relations,

$$p(x,y) \triangleq p_{\{x,y\}}(x) \tag{3}$$

is used in the place of $p_\xi(x)$ in *Eq.*(2),where $\xi = \{x, y\}$.

2.2 Random Utility Models

Assume that an information structure is given and the choice behavior is restricted with it. Let U be a function defined on the outcome set X (A) such that

$$P_r[U(x) \ge U(y);\ x,y \in \xi \subseteq X]$$
$$\triangleq \int_{-\infty}^{\infty} P_r[U(x)=t, U(y) \le t; x,y \in \xi \subseteq X]dt. \tag{4}$$

The values of the function U is a vector of random variables. The P_r is called the *preference probability.* A set of the preference probabilities defines the *random utility model* such that

$$p_\xi(x) = P_r[U(x) \ge U(y); x,y \in \xi \subseteq X], \tag{5}$$

where the U is the random utility function.

In the binary preference relations,

$$p(x,y) = P_r[U(x) \ge U(y)]. \tag{6}$$

The value of the preference probability P_r represents a value of the possible distribution of the preference values to be assessed by the anonymous IDM in group decision making.

The preference independence rule for the probabilistic choice behavior is presented in terms of the probability independence rule. Assume the preference evaluation as the random variables be independent of each other. Then the random utility model is rewritten from *Eq.*(4) by the probability rule as

$$P_r[U(x) \geq U(y);\; x, y \in \xi \subseteq X] \triangleq \int_{-\infty}^{\infty} P_r[U(x)=t] \prod_{y \in \xi - \{x\}} P_r[U(y) \leq t] dt. \quad (7)$$

The irrelevance rule for the probabilistic choice behavior is presented in terms of the choice probability as

$$p_Y(x) = p_\xi(x \mid Y), \text{ for all } x \in Y \subseteq \xi \subseteq X, \quad (8)$$

These rules correspond to the equivalent rules in the "algebraic" decision theory.

The random utility function in group decision making is a latent concept in the random utility model, where it is not necessary to reveal its function forms. We shall still examine, however, their properties to be consistent with the probabilistic choice model.

3. Stochastic Expected Utility Models

Consider the probabilistic choice model for two alternative gambles i and j. Define the strong expected utility values for alternative gambles i and k by

$$\bar{v}^{(i)} \triangleq \sum_j^n \pi_j^i v(x_j) \text{ and } \bar{v}^{(k)} \triangleq \sum_j^n \pi_j^k v(x_j). \quad (9)$$

Then, using $\bar{u} = \log \bar{v} + b$, the strict expected utility model can be rewritten with the logistic distribution function $\phi(t) = 1/(1+e^{-t})$, such that

$$p(\pi^i, \pi^k) = \frac{\sum_{j=1}^n \pi_j^i v(x_j)}{\sum_{j=1}^n \pi_j^i v(x_j) + \sum_{j=1}^n \pi_j^k v(x_j)} = \frac{1}{1 + \frac{\sum_{j=1}^n \pi_j^k v(x_j)}{\sum_{j=1}^n \pi_j^i v(x_j)}} = \frac{1}{1 + \frac{\bar{v}^{(k)}}{\bar{v}^{(i)}}} = \frac{1}{1+\exp[-\{\bar{u}^{(i)} - \bar{u}^{(k)}\}]} = \phi[\bar{u}^{(i)} - \bar{u}^{(k)}] \quad (10)$$

where $\bar{u}^{(i)}$ and $\bar{u}^{(k)}$ are the alternative gamble evaluations for the "natural" event θ_j, $j = 1, \ldots, n$, with the strong utility functions. The difference of the gamble evaluation can be assessed, by assuming the risk neutral attitudes, as a positive linear function of the expectation $\bar{\pi}^i(x)$ of the most preferable gamble π^i such that

$$\phi[\bar{u}^{(i)}(x) - \bar{u}^{(k)}(x)] = \phi[\gamma(\bar{\pi}^i(x)) + \delta] \quad (11)$$

Then we can use the logistic distribution functions in the evaluation of the choice probability $p(\pi^i, \pi^k)$.

4. Probabilistic Value Tradeoffs and Multiple Risk Evaluation

In multiple risk assessment, the incomplete information structure should be taken into account and the preference structure for the alternative gambles should be examined. For this purpose, a composite risk function defined on the alternative gambles should be evaluated. We discuss the probabilistic value tradeoffs between alternative gambles for its construction.

Let $A, B, C, \ldots$ denote the alternative gambles which have the different "natural" probability assignment for a prospect depending on the diversified incomplete information structure ϖ^A. Define the gamble $\pi^A = (\pi_1^A, \cdots, \pi_n^A)$, $\pi^A \subseteq \Pi$, on a value set $x = (x_1, \ldots, x_m)$ of the multiple attributes. The *risk evaluation function* $\Re$ *for a gamble* is defined on the expectation $E\pi^A$ of a gamble π^A such that

$$\Re(E\pi^A(\varpi^A)) \triangleq \Re[(\pi^A_1, x_1 \mid \theta^A_1(\varpi^A)), (\pi^A_2, x_2 \mid \theta^A_2(\varpi^A)), \ldots, (\pi^A_n, x_n \mid \theta^A_n(\varpi^A))], \quad (12)$$

where $x_j = (x_{1j}, \ldots, x_{mj})$, $j = 1, \ldots, n$, is a multiattribute value set to be obtained as the outcome of the occurrence of an uncertain event θ_j. Define the expectation for a multiattribute gamble as

$$E\pi^A \triangleq \frac{1}{m} \sum_{i=1}^m \sum_{j=1}^n \pi_j^A x_{ij}, \quad (13)$$

or alternatively if it is preferable,

$$E\pi^A \triangleq \sqrt[m]{\prod_{i=1}^m \left(\sum_{j=1}^n \pi_j^A x_{ij}\right)} \quad \text{for } A, B, C, \ldots \quad (14)$$

The probabilistic value tradeoffs are defined on the expectations of the alternative gambles for the multiple attributes as

$$\Delta E\pi^A / \Delta E\pi^B \quad \text{for } A, B, C, \cdots. \quad (15)$$

A composite risk evaluation function defined with the alternative gambles can be derived, via the probabilistic value tradeoffs.

Note that, the evaluation of the "natural" probability and the risk tradeoffs for the alternative gamble prospects is the technical problem. This work does not present the preference evaluation problem for the attributes. In the construction of the composit risk function, the function form of the component risk function is not necessary to be evaluated as the revelation of the preference structure for the attributes in group decision making. Only considerable risk attitudes and their degree of the strongness should be assumed, such as in a linear form.

(* The full paper is available on request.
E-mail: a50202@sakura.kudpc.kyoto-u.ac.jp)