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Pricing Strategies for Balking Queues

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1. Introduction A birth-death type queue is considered where each customer brings a random valued job to be processed in the system. Each arriving customer joins the queue only if the delay cost is less than the job value, otherwise the customer leaves the system. The delay cost does not have to be linear but an increasing function of the number of customers in the system. We analyze how the distribution of job value would affect the social benefit of the system which is defined as the expected net value processed in the system per unit of time. Since customers rushing to the system could cause a huge congestion which would increase the delay cost, the stochastically higher job value does not always produce the higher social benefit. However we show that this monotonic relationship holds in the exponential job value case. We also investigate a generalized pricing strategy.

2. The Model We consider a single server queue with exponential server with service rate $\mu > 0$. Customers arrive at the system in a Poisson stream with parameter $\lambda > 0$. The job value for the i-th arriving customer is a random variable which is denoted by V_i where V_i are i.i.d. with a common distribution function $F(x) = \text{Prob}[V_i \leq x]$. The corresponding generic random variable is denoted by V.

Customers observe the system status on their arrival and decide to submit their jobs, or to leave (balk) without submitting their jobs by comparing their job values with the expected delay costs. The expected delay cost is possibly nonlinear in the expected sojourn time. Let c_n be the expected delay cost for an arriving customer who observes n customers in the system. We assume that c_n is positive and a non-decreasing function of n, i.e., $0 < c_0 \leq c_1 \leq c_2 \leq \cdots$, and that $\lim_{n\to\infty} c_n = \infty$. The steady state probability of n customers in the system, e_n, is easily obtained. We define m_s by

$$m_s = \lambda \sum_{n=0}^{\infty} e_n \int_{c_n}^{\infty} x dF(x) - \sum_{n=0}^{\infty} e_n \lambda_n c_n = \lambda \sum_{n=0}^{\infty} e_n \int_{c_n}^{\infty} \bar{F}(x) dx, \tag{1}$$

i.e., m_s is the expected sum of the net job value processed in the system per unit of time. We take a viewpoint that m_s is the public good which should be optimized, and we call m_s the social benefit of the system.

3. Effect of The Higher Job Value on The Social Benefit

Theorem 1 *Suppose that all users have a constant job value v such as $c_0 < v$, then the monotonicity of $m_s(v)$, in general, is not satisfied.*

Theorem 2 *Suppose that users in System-1 and users in System-2 have exponential job values V_1 and V_2 respectively. Let $\bar{F}_1(x) = P[V_1 > x] = e^{-\eta_1 x}$ and $\bar{F}_2(x) = P[V_2 > x] = e^{-\eta_2 x}$. If $V_1 \leq_{st} V_2$, then $m_s(\eta_1) \leq m_s(\eta_2)$.*

4. The (k,p) Price Strategy We consider a socially optimal policy introducing a generalized price strategy. The price p_n is charged for arriving customers who find n customers in system, if they join the system. Consequently this kind of price charge accelerates them to balk, and then it affects the effective arrival rates. We define the social benefit $m_s(p)$ for this model in the same manner as the no price model, that is,

$$
\begin{aligned}
(2) \qquad m_s(\boldsymbol{p}) &\equiv \lambda \sum_{n=0}^{\infty} e_n(\boldsymbol{p}) \int_{c_n+p_n}^{\infty} x dF(x) - \sum_{n=0}^{\infty} e_n(\boldsymbol{p}) \cdot \lambda_n(p_n) \cdot c_n \\
&= \lambda \sum_{n=0}^{\infty} e_n(\boldsymbol{p}) \int_{c_n+p_n}^{\infty} \bar{F}(x) dx + \sum_{n=0}^{\infty} e_n(\boldsymbol{p}) \cdot \lambda_n(p_n) \cdot p_n
\end{aligned}
$$

Particularly, we deal with the (k,p) price strategy which is written as $p_n = 0$ for $n \leq k-1$ and $p_n = p$ for $n \geq k$. This type of price strategy includes the following strategies.

- The free access strategy ($p = 0$ or $k = \infty$)
- The complete charge strategy ($k = 0$)
- The N-gated strategy ($p = \infty$ and $k = N$)

The next theorem guarantees a positive price p which strictly increases the social benefit in the (k,p) price strategy for an arbitrary integer $k \geq 0$.

Theorem 3 *Suppose that users have random job value V with the distribution function $F(x) = 1 - e^{-\eta x}$. Then, for an arbitrary fixed integer $k \geq 0$, there exists a positive optimal price $p = p^*$ which maximize the social benefit $m_s(k,p)$ with the (k,p) price strategy.*

Theorem 4 *Suppose that all users have a constant job value v and assume $0 < \rho < 1$. Define an integer $N(k,p)$ as*

$$c_{N(k,p)} + p_{N(k,p)} < v \leq c_{N(k,p)+1} + p_{N(k,p)+1},$$

and define a function $g(i)$ as

$$g(i) \equiv (1-\rho^{i+2}) \sum_{n=0}^{i+1} \rho^n (v - c_n) - (1-\rho^{i+3}) \sum_{n=0}^{i} \rho^n (v - c_n),$$

and N is the critical number such that $c_N < v \leq c_{N+1}$.

If $g(N-1) \geq 0$ then the (k,p) strategy such that $N(k,p) = N$ is optimal.

If $g(N-1) < 0$ then there exists the unique number i^* $(0 < i^* < N-1)$ such that $g(i-1) > 0$ and $g(i) < 0$, and the (k,p) strategy such that $N(k,p) = i^*$ is optimal.

References

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