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離散時間 M/G/1 型待ち行列の稼働期間モーメント計算アルゴリズム

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Moment Calculating Algorithm of Busy Period of Discrete-Time M/G/1 Type Queue

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Abstract

The discrete-time single server queues with finite buffer are often used to evaluate the performance of ATM switches and multiplexers. In many cases, the behavior of such queues is governed by the skip-free Markov chain in upper direction with the upper Hessemberg transition probability matrix. In this study, we call such a queue "M/G/1 type queue".

Many researchers have studied the performance measures concerning to the the discrete-time M/G/1 type queue with finite buffer. Among such performance measures, we focus on the moments of busy period of the discrete-time single server queue with finite buffer.

In Ref.[4], the authors provide the algorithm to calculate the distribution of the busy period of M/G/1 type queue which serves a customer (cell) every slot if any. However, much computational effort is necessary since all sample paths are traced to calculate the distributions of busy periods in this algorithm. Further, it is necessary to calculate the probability distribution over enough length of busy period in order to obtain the accurate moments of the busy period.

In Ref.[3], the author shows a set of recursive relations for the LST (Laplace Stieltjes Transform) of busy period for the M/G/1/K queue. By differentiating the recursive equations, we can obtain the higher moments of busy period. Note that M/G/1/K queue has a structure of infinitesimal generator of the process similar to that of transition probability matrix of discrete-time M/G/1 type queue with finite buffer.

In this study, we propose an algorithm to calculate the moments of the busy period of the discrete-time M/G/1 type queue with finite buffer, where L cells including served one can exist in the system at most. Our algorithm is based on the extended version[2] of Neuts's algorithm for M/G/1 type queue with infinite buffer[1].

In our model, time is slotted, and the state transition occurs every slot if any. Further, the arrival and service processes have S states together and they are possible to depend on the state of the level. (Here after, we call the level at the observation point "*phase*".) Thus, our model has the $(L+1)S \times (L+1)S$ transition probability matrix

$$\boldsymbol{Q} = \begin{bmatrix} \boldsymbol{A}_{0,0} & \boldsymbol{A}_{0,1} & \cdots & \boldsymbol{A}_{0,L-1} & \boldsymbol{A}_{0,L} \\ \boldsymbol{A}_{1,0} & \boldsymbol{A}_{1,1} & \cdots & \boldsymbol{A}_{1,L-1} & \boldsymbol{A}_{1,L} \\ \boldsymbol{O}_S & \boldsymbol{A}_{2,0} & \cdots & \boldsymbol{A}_{2,L-2} & \boldsymbol{A}_{2,L-1} \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \boldsymbol{O}_S & \cdots & \boldsymbol{O}_S & \boldsymbol{A}_{L,0} & \boldsymbol{A}_{L,1} \end{bmatrix}.$$

As shown in Ref.[2] we have for this model

$$G^{(l)}(z) = z\left[I_S - \overline{A}_{l,l}(z)\right]^{-1} A_{l,0},$$

where $G^{(l)}(z)$ is the z-transform matrix of the first passage time from the level l to $l-1$, and

$$\overline{A}_{l,l'}(z) \triangleq z \sum_{m=l'-l}^{L-l} A_{l,m+\min\{l,1\}} \prod_{j=0}^{l+m-l'-1} G^{(l+m-j)}(z).$$

With the z-transform of the busy period, $b(z) \triangleq \sum_{k=1}^{\infty} \Pr\{B=k\}z^k$ for $|z| \leq 1$, we have

$$b(z) = \frac{x_0 Y(z) e_S}{x_0(I_S - A_{0,0})e_S},$$

where

$$Y(z) = \sum_{l=1}^{L} A_{0,l} \prod_{j=0}^{l-1} G^{(l-j)}(z).$$

Note that the following relationship holds:

$$K(z) = \overline{A}_{0,0}(z)) = z(A_{0,0} + Y(z)).$$

We therefore need the kth differential matrix of $K(z) = \overline{A}_{0,0}(z)$ with respect to z to calculate the kth moments of busy period. To do so, we propose a set of recursive relations for the derivatives of $G(z)$. By substituting $z=1$ for the derivatives of $G(z)$, we can obtain the higher moments of busy period of our queueing model. Note that this algorithm finishes in finite procedures, and further is applicable to the M/G/1 type queue with finite buffer which does not finish the service in one slot.

References

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