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Finding a Common Weight Vector of DEA Based on Bargaining Game

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1 Introduction

A large number of studies on DEA have been made and developed since it was first proposed in [1], as confirmed in Seiford's paper.

DEA is a mathematical programming approach to assessing relative efficiencies within a group of DMUs. An important outcome of such an analysis is a set of virtual multipliers or weights accorded to each (input or output) factor taken into account. These sets of weights are, typically, different for each of the participating DMUs.

There had defined the cross-efficiency of DMU_j as measured by DMU_o as the ratio of weighted output to weighted input obtained when we use the input and output levels of DMU_j and the input and output weights derived for DMU_o[2]. It is widely known that the weights are not always uniquely determined. The mutual evaluation information which, we introduced, are calculated from the weights are not always determined uniquely neither. In addition to, there may be not the weights of the cross-efficiency in feasible set.

Here, we proposes a method for determining a common weight vector of DEA based on bargaining game. Furthermore, this paper introduces an example of the proposed method to the productivity analysis of Japanese electric power industries in order to demonstrate the effectiveness of the method.

2 DEA Model and Cross-Efficiency

There are various descriptions about DEA. For the sake of uniformity of symbols and expressions, we would like to describe in the same manner with [3, 4].

The cross-efficiency was proposed by Sexton, Silkman and Hogan[2]. In their paper, they had defined the cross-efficiency of DMU_j as measured by DMU_o as the ratio of weighted output to weighted input obtained when we use the input and output levels of DMU_j and the input and output weights derived for DMU_o. Mathematically, the cross-efficiency is the ratio of the sums on the left side of constraint j in the (DEA) problem for DMU_o: $E_{oj} = \sum_{r=1}^{s} u_{ro}^* y_{rj} / \sum_{i=1}^{m} v_{io}^* x_{ij}$.

The accommodation total efficiency was proposed by Sugiyama and Yamada[3, 4], and we can say the accommodation total efficiency is general form for the cross-efficiency.

3 A Common Weight Vector by Bargaining Game Approach

3.1 Bargaining Game

A bargainig game is a pair (S, d). The point d is the disagreement outcome or disagreement point results. The set S is called the feasible set. The Kalai-Smorodinsky bargaining solution is the only solution on fesible set satisfying *Pareto-opimality, symmetry, scale transformation convariance* and *individual monotoniicity*[5].

[The Kalai-Smorodinsky bargaining solution], $K(S)$: $K(S)$ is the maximal point of S on the segment connecting the origin to $a(S)$, *the ideal point of* S, defined by $a_k(S) \equiv \max\{\hat{x}_k | \hat{x} \in S\}$ for all k.

3.2 The feasible set S of bargaining game on DEA

Now, we will generalize the feasible set. The feasible set in corralated pure strategy is expressed as follows:

$$S^{\mathrm{p}} = \left\{ \eta \in \mathrm{R}^n \;\middle|\; \begin{array}{l} \eta_j = \dfrac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \quad (j = 1, \ldots, n), \\ \dfrac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 \ (j = 1, \ldots, n), \\ u_r \geq 0 \ (r = 1, \ldots, s), \\ v_i \geq 0 \ (i = 1, \ldots, m) \end{array} \right\}.$$

The feasible set in corralated mixed strategy is expressed as follows: $S^{\mathrm{m}} =$

$$\left\{ \eta' \in \mathrm{R}^n \;\middle|\; \begin{array}{l} \eta' = \mu_1 \eta'_1 + \cdots + \mu_o \eta'_o + \cdots + \mu_n \eta'_n, \\ \sum_{o=1}^{n} \mu_o = 1, \\ \eta'_o = (\eta'_{1o}, \ldots, \eta'_{jo}, \ldots, \eta'_{no})^{\mathrm{T}} \ (o = 1, \ldots, n), \\ \eta'_{jo} = \dfrac{\sum_{r=1}^{s} u_{ro} y_{rj}}{\sum_{i=1}^{m} v_{io} x_{ij}} \quad (j = 1, \ldots, n \,;\, o = 1, \ldots, n), \\ \dfrac{\sum_{r=1}^{s} u_{ro} y_{rj}}{\sum_{i=1}^{m} v_{io} x_{ij}} \leq 1 \ (j = 1, \ldots, n \,;\, o = 1, \ldots, n), \\ u_{ro} \geq 0 \ (r = 1, \ldots, s \,;\, o = 1, \ldots, n), \\ v_{io} \geq 0 \ (i = 1, \ldots, m \,;\, o = 1, \ldots, n) \end{array} \right\}.$$

4 Application Example

This section introduces an example which is the productivity analysis of Japanese electric power industries by applying the proposed approach. This example was given in [3, 4].

4.1 Analysis and Evaluation

First, DEA is conducted concerning nine electric power companies as DMUs.

Second, we calculate from a common weight vector of DEA based on the Kalai-Smorodinsky bargaining solution. Let the ideal point of S be an each DMU's DEA-efficiency score. It can establish the disagreement point d as various kinds of point, but establishes it as origin in this paper. Table 1 and Table 2 indicate the results, i.e., efficiencies for each DMU and a common weight vector, obtained from the Kalai-Smorodinsky barigaining solution.

Table 3 indicates the results obtained from the Kalai-Smorodinsky barigaining solution and conventional approaches, respectively. Here, the accommodation total efficiencies and the cross-efficiencies of each DMU were given in [3, 4]. And, the cross-efficiencies of each DMU were calculated by using the weights determined uniquely in [3, 4].

Table 1: The Kalai-Smorodinsky Barigaining Solution of Each Electric Power Company

	The Kalai-Smorodinsky Barigaining Solution		DEA-efficiency
	corralated pure strategy	corralated mixed strategy	(θ_o^*)
Hokkaido	0.8411	0.8411	1.0000
Tohoku	1.0000	1.0000	1.0000
Tokyo	1.0000	1.0000	1.0000
Chubu	1.0000	1.0000	1.0000
Hokuriku	0.8082	0.8082	0.9245
Kansai	0.9928	0.9928	0.9928
Chugoku	0.9716	0.9716	0.9906
Shikoku	0.8071	0.8071	0.8430
Kyushu	0.8863	0.8863	0.9560

Table 2: A Common Weight Vector

	$\bar{v}_1$	$\bar{v}_2$	$\bar{v}_3$	$\bar{u}_1$	$\bar{u}_2$
corralated pure strategy	0.6779	0.0000	2.3778	2.3584	0.5912
corralated mixed strategy	0.6779	0.0000	2.3778	2.3584	0.5912

Table 3: Efficiencies of Each Electric Power Company

	The Kalai-Smorodinsky Barigaining Solution	The Accommodation Total Efficiency	The Cross-Efficiency (by the weights determined uniquely)
Hokkaido	0.8411	0.9155	0.8884
Tohoku	1.0000	1.0000	0.9757
Tokyo	1.0000	0.9658	0.9419
Chubu	1.0000	0.8643	0.8476
Hokuriku	0.8082	0.7244	0.7120
Kansai	0.9928	0.8500	0.8321
Chugoku	0.9716	0.9018	0.8824
Shikoku	0.8071	0.7879	0.7689
Kyushu	0.8863	0.8893	0.8655

5 Conclusion

We introduced a method for determining a common weight vector of DEA based on the Kalai-Smorodinsky bargaining solution. In this paper, we introduced an example of the proposed method to the productivity analysis of Japanese electric power industries in order to demonstrate the effectiveness of the method.

References

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