可能性理論により新聞売り子問題について

01991735 香川大学 郭 沛俊 GUO Peijun

1. Possibilistic model for Newsboy problem

Consider a retailer who sells a short life cycle, or single-period new product. The retailer orders q units before the season at the unit wholesale price W. Then demand d is observed, and the retailer sell units (limited by the supply q and the demand d) at unit revenue R ($R>W$). Any excess units can be salvaged at the unit salvage value S. For example, excess units can be sold at a reduced price $S<W$. Prior to the selling season, demand is uncertain. However the retailer can know plausible information of demand represented by a possibility distribution. The profit function of the retailer is as follows:

$$r = R\min(d,q) + S(q-d)^{+} - Wq, \tag{1}$$

where q is a decision variable and d is governed by a possibility distribution $\pi(d)$ given by experts to reflect plausible information on the demand and a^{+} is

$$a^{+} = \begin{cases} a\,; a>0 \\ 0\,; a\le 0 \end{cases}. \tag{2}$$

Suppose that the possibility distribution for demand is characterized by the following triangular function

$$\pi(d) = \begin{cases} 1-\dfrac{d_c-d}{d_c-d_l} & ; \quad d<d_c \\ 1 & ; \quad d=d_c \\ 1-\dfrac{d-d_c}{d_u-d_c} & ; \quad d>d_c \\ 0 & ; \quad otherwise \end{cases} \tag{3}$$

Definition 1. For a given supply q, the utility function $u(d,q)$ is given as follows:

$$u(d,q) = \begin{cases} \left(\dfrac{q}{d_u} - C(q)\right)\dfrac{d-d_l}{q-d_l} + (1+\varepsilon)C(q) = \dfrac{R-S}{(R-W)d_u}(d-d_l) + (1+\varepsilon)C(q); & d\le q \\ \dfrac{q}{d_u} + \varepsilon C(q); & d>q \end{cases}, \tag{4}$$

$$C(q) = \frac{(R-S)d_l - (W-S)q}{(R-W)d_u}. \tag{5}$$

Theorem 1. Optimal supply q_o^{*} based on the optimistic criterion defined in [1] is

$$q_o^{*} = \frac{B2}{B1}, \tag{6}$$

where

$$B1 = (2d_u - d_c)(R-W) - \varepsilon(d_u - d_c)(W-S),\quad B2 = d_u^2(R-W) - \varepsilon(d_u-d_c)d_l(R-S),\quad 0\le\varepsilon<\frac{R-W}{W-S}.$$

Theorem 2. Optimal supply q_p^{*} based on the pessimistic criterion defined in [1] is

$$q_p^{*} = \frac{B4}{B3}, \tag{7}$$

where

$$B3=(d_u+d_c-d_l)(R-W)+\varepsilon(d_l-d_c)(W-S),\quad B4=d_u d_c(R-W)-\varepsilon d_l(d_c-d_l)(R-S),\quad 0\le\varepsilon<\frac{R-W}{W-S}.$$

2. Possibilistic model for supply contract: real option

Ahead of season, the retailer buys q call options at unit cost T. Each call option gives the retailer the right to buy a unit of the product at the unit exercise price P after the retailer observes the demand. If the demand d is not less than q, than retailer can buy q units of product at the exercise price P ($P<W$), else buy d units of product at the price P. The profit of the retailer, r, is given by the following expression:

$$r=(R-P)\min(d,q)-Tq \tag{8}$$

Definition 2. For a given supply q, the utility function $u(d,q)$ is given as follows:

$$u(d,q)=\begin{cases}(\frac{q}{d_u}-V(q))\frac{d-d_l}{q-d_l}+(1+\varepsilon)V(q)=\frac{R-P}{(R-P-T)d_u}(d-d_l)+(1+\varepsilon)V(q); & d\le q\\ \frac{q}{d_u}+\varepsilon V(q); & d>q\end{cases}, \tag{9}$$

$$V(q)=\frac{(R-P)d_l-Tq}{(R-P-T)d_u}. \tag{10}$$

Theorem 3. Optimal supply q^*_{ro} for real option based on the optimistic criterion is

$$q^*_{ro}=\frac{B6}{B5}, \tag{11}$$

where

$$B5=(2d_u-d_c)(R-P-T)-\varepsilon(d_u-d_c)T,\quad B6=d_u^2(R-P-T)-\varepsilon(d_u-d_c)d_l(R-P),\quad 0\le\varepsilon\le\frac{R-P-T}{T}.$$

Theorem 4. Optimal supply q^*_{rp} for real option based on the pessimistic criterion is

$$q^*_{rp}=\frac{B8}{B7}, \tag{12}$$

where

$$B7=(d_u+d_c-d_l)(R-P-T)+\varepsilon(d_l-d_c)T,\quad B8=d_u d_c(R-P-T)+\varepsilon d_l(d_l-d_c)(R-P),\quad 0\le\varepsilon\le\frac{R-P-T}{T}.$$

References

[1] Dubois, D., Nguyen, H. T. and Prade, H., Possibility theory, probability and fuzzy sets: Misunderstanding, Bridges and gaps, Fundamentals of Fuzzy Sets (Dubois, D and Prade, H. eds.), Handbook of Fuzzy Sets, Kluwer Avademic Publication, 2000, 343-438.

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