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An exact quantization method for the design of linear phase FIR filter using Semi-Infinite Linear Programming

Rika Ito* Kenji Suyama* Ryuichi Hirabayashi*

1 Introduction

The purpose of the paper is to propose a new design method of FIR filters with discrete coefficients considering optimality. The design methods of FIR filters with discrete coefficients have been widely researched[1]~[4]. However, the optimality of the solution has not be assured because of the finite constraints. In our proposed method, the design problem of FIR filters is formulated as a Mixed Integer Semi-Infinite Linear Programming problem (MISILP), which can be solved by a branch and bound technique. On each node of the branching tree, it is necessary to solve Semi-Infinite Linear Programming problem (SILP)[5]. Then it is possible to obtain the optimal discrete coefficients, and the optimality of the obtained solution can be guaranteed. It was confirmed that optimal coefficients of linear phase FIR filter with discrete coefficients could be designed in reasonable computational time with sufficient precision based on the results of computational experiments.

2 Problem Formulation

The transfer function of an FIR filter with length $N+1$ is denoted as

$$H(z)=\sum_{k=0}^{N} h_k z^{-k}. \tag{1}$$

When h_k, $k=0,1,\ldots,N$ is the even symmetric impulse response and, N is an even number, the linear phase characteristic with $N/2$ delay is achieved. Then, the magnitude response $H(\omega)$ can be expressed as

$$H(\omega)=\sum_{n=0}^{N} a_n \cos n\omega. \tag{2}$$

Suppose, a desired response $D(\omega)$ is given as follows

$$D(\omega)=\begin{cases} K, & 0\le\omega\le\omega_p, \\ 0, & \omega_s\le\omega\le\pi. \end{cases} \tag{3}$$

Where K is a scaling factor, ω_p is the passband cutoff frequency, and ω_s is the stopband cutoff frequency, respectively. Then, the optimization problem to approximate $H(\omega)$ to $D(\omega)$ in a min-max sense can be written as

$$\min_{a_0,\cdots,a_N}\ \max_{\omega\in\Omega} |D(\omega)-H(\omega)|. \tag{4}$$

$$\Omega=[0,\omega_p]\cup[\omega_s,\pi].$$

If we introduce a new variable δ' that corresponds to the L_∞-approximation error, and assume that coefficients a_n $(i=0\ldots N)$ are limited to discrete coefficients of p bit, it is easy to convert the above min-max problem into the MISILP as follows.

$$\begin{array}{lrl} \min & \delta & \\ \text{sub.to} & \tilde{H}(\omega)+\delta & \ge 2^p D(\omega), \omega\in\Omega, \\ & -\tilde{H}(\omega)+\delta & \ge -2^p D(\omega), \omega\in\Omega \\ & x_0,\ldots,x_N & \ge -2^p, \\ & -x_0,\ldots,-x_N & \ge -(2^p-1), \\ & x_i\in\mathbf{Z}, & i=0,\ldots,N, \end{array} \tag{5}$$

where,

$$\begin{aligned} \tilde{H}(\omega) &= 2^p H(\omega) && (6)\\ &= \sum_{n=0}^{N} 2^p a_n \cos n\omega && (7)\\ &= \sum_{n=0}^{N} x_n \cos n\omega, && (8)\\ x_i &= 2^p a_i,\ i=0,\ldots,N, && (9)\\ \delta &= 2^p\delta'. && (10) \end{aligned}$$

3 A New Design Method using Semi-Infinite Linear Programming

Our aim is to solve MISILP(5), but it is impossible to solve it directly. Hence, we solve SILP ignoring the integer constraints. However, since SILP is a continuous optimization problem, an optimal solution obtained is not always an integer solution. A standard technique for solving this difficulty is to exploit the B & B technique.

If there are some $\bar{x}_i$'s that are not integers, then select one non-integer variable x_j and generate two subproblems, which one has an additional constraint $-x_j\ge-\lfloor\bar{x}_j\rfloor$ and the other has an additional constraints $x_j\ge\lceil\bar{x}_j\rceil$. Notice here, that the two generated subproblems are also SILP and can be solved by Three Phase method. We can continue this procedure and call this process as branching process.

*Science University of Tokyo, Hirabayshi Laboratory, Faculty of Engineering,Japan, E-mail:{rika, suyama, hira}@ms.kagu.sut.ac.jp,

If we continue the branching process, then after finite iterations, we can obtain an integer solution. The obtained integer solution is an optimal solution for the subproblem and a feasible solution for MISILP (5), but might not be optimal for MISILP(5). However, we can use the objective function value that corresponds to the integer solution as an upper bound for MISILP (incumbent value) since we can fathom subproblems that have the optimal value greater than or equal to the upper bound. This is true, because, if we add some additional constraints, the optimal value of the subproblem becomes always bigger. The process that we fathom all subproblems which have greater optimal value than the incumbent value is called the bounding process.

4 Computational Experiments

We executed some computational experiments to certify the performance of the proposed filter design method. We set $\omega_p = 2/5\pi, \omega_s = 4/7\pi$. Two kinds of computational experiments were performed.

(a)The scaling factor is fixed to $K = 1$. The bit length p was set from 3 to 10 with pitch 1, and the filter order was fixed to $N = 3, 4, \ldots, 20$ for each value of p.

(b)We fixed $p = 6, 7, 8$ and $N = 9, 10, 11, 12$. Then, the scaling factor K was changed from 0.5 to 2.0 with pitch 0.1.

The result of experiment (a) for $N = 12$, $p = 3, \ldots, 10$ is shown in figure 1. It was shown that the optimal value decreased slowly for p over 7bit, on the other hand, the computational time increased rapidly. Therefore, we can attain fast with the enough approximation with only 6bit word-length.

Figure 2 shows the result of experiment (b) for $p = 4$, $N = 12$, $K = 0.5 \ldots 2.0$. From this results it was indicated that the appropriate value of K is from 1.0 to 1.5.

Figure 3 shows the (A)magnitude response for $p = 4$, $N = 12$, $K = 1$ and the (B)magnitude response using the coefficients which are simply rounded to 4bit.

5 Conclusion

In this paper, we proposed a new design method for FIR filters with discrete coefficients using B & B technique. In this method, by formulating the FIR filter design problem to MISILP, we can guarantee the optimality of the obtained filter coefficients. The computational experiments showed that the proposed method performed the enough approximation even for only 6bit word-length with the reasonable computational time.

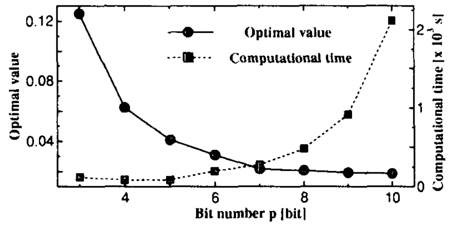

Figure 1: Experimental result for $p = 3 \ldots 10, N = 12, K = 1$

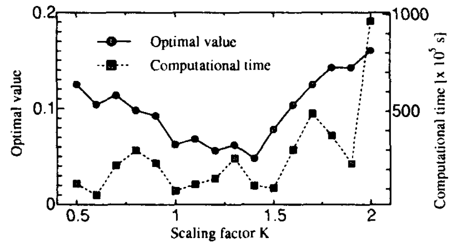

Figure 2: Experimental result for $p = 4, N = 12, K = 0.5 \ldots 2.0$

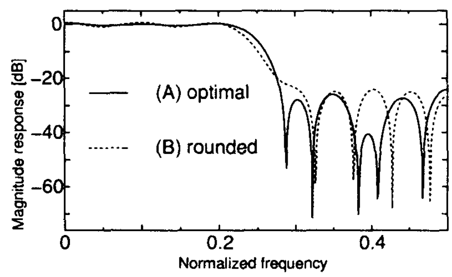

Figure 3: Magnitude response for (A) $p = 4, N = 12, K = 1$ and (B) with coefficients rounded to $p = 4$

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