Note on Equitable Round-Robin Tournaments

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1. Introduction

A round-robin tournament is a popular tournament type for sports competition, and often held with stadium assignment. In the tournament, every team has its home stadium. In this paper, we consider a round robin tournament with stadium assignment consisting of $2N$ teams and $2N-1$ slots (Figure 1).

		1	2	3	4	5
1	:	6	$\boxed{3}$	$\boxed{5}$	2	4
2	:	$\boxed{4}$	6	$\boxed{3}$	$\boxed{1}$	5
3	:	$\boxed{5}$	1	2	$\boxed{4}$	$\boxed{6}$
4	:	2	5	$\boxed{6}$	3	$\boxed{1}$
5	:	3	$\boxed{4}$	1	$\boxed{6}$	$\boxed{2}$
6	:	$\boxed{1}$	$\boxed{2}$	4	5	3

Figure 1. Round-Robin Tournament

In Figure 1, every column shows games on a slot and every row shows opponents of a team. Each game is held at one of the home grounds of the corresponding pair of teams. If a game between teams i and j is played at the home ground of i, the game is a *home-game* for i and an *away-game* for j. Boxed cells in Figure 1 denote away-games of the team corresponding to the row.

Stadium assignment has a serious effect on a result of the tournament. Therefore, a tournament organizer would like to construct stadium assignment in order to satisfy required conditions. In this paper, we consider stadium assignment in terms of fairness to all teams. We state several conditions for "good" stadium assignment, and show that the number of stadium assignment satisfying all the conditions is very small.

2. Home-Away Table (HAT)

In this section, we introduce *home-away table* (HAT), which expresses stadium assignment. In HAT, a home-game of the team corresponding to the row is represented by 'H', and away-game by 'A'. *Home-away pattern* (HA-pattern) is a row of HAT. Figure 2 is HAT corresponding to Figure 1.

		1	2	3	4	5
1	:	H	A	$\underline{A}$	H	$\underline{H}$
2	:	A	H	A	$\underline{A}$	H
3	:	A	H	$\underline{H}$	A	$\underline{A}$
4	:	H	$\underline{H}$	A	H	A
5	:	H	A	H	A	$\underline{A}$
6	:	A	$\underline{A}$	H	$\underline{H}$	$\underline{H}$

Figure 2. HAT corresponding to Fig. 1

If there exists a schedule corresponding to a given HAT, the HAT is called *feasible*. Note that there are *infeasible* HAT, i.e., HAT which does not correspond to any schedules.

When there are consecutive 'A's or 'H's in a particular HA-pattern, we say the HA-pattern has a *break*. As above, we express a break by a line under latter 'A' or 'H' in HA-pattern.

In general, players of a team like a home-game rather than an away-game. HA-pattern including consecutive 'A's, e.g. "A$\underline{AAA}$HAH", is not desirable for any teams. HA-pattern which contains consecutive 'H's is not preferable neither. The organizer usually wants to reduce the number of breaks in HAT.

It is already known that existence of HAT with $2N$ breaks[1]. We call such HAT *equitable* HAT. Henceforth, we consider equitable HAT since equitable HAT have good properties in terms of fairness to all teams[2].

In addition, we require that HAT must satisfy following conditions for a practical reason. In a round-robin tournament consisting of $2N$ teams

($N \geq 3$), games on slot 1 and 2 and games on slot $2N-2$ and $2N-1$ are relatively important for the fans. Thus, the organizer does not prefer HAT which contains HA-pattern whose slot 1 and 2 ($2N-1$ and $2N$) are 'A'. If HAT satisfies that at least one of the first and second elements of each row (HA-pattern) is 'H', we say that the HAT satisfies *opening condition*. Similarly, we define *closing condition*.

The number of equitable HAT which meet opening and closing condition is ${}_{2N-4}C_N$[2] (Table 2). However, the number is obtained by taking both feasible and infeasible HAT into account. In the next section, we perform computational experiments to enumerate all the feasible HAT.

3. Computational Experiments

We formulate the problem to decide the feasibility of given HAT as IP[2]. Given HAT, we define the following notations. Let P be the set of all the triplets (i,j,s) satisfying that the cell of the given HAT corresponding to team i and slot s is 'A' and the cell of team j and slot s is 'H'. We introduce a 0-1 variable x_{ijs} for each triplets $(i,j,s) \in P$ whose value is equal to 1 if and only if team i and j plays a game on slot s. Then the problem for checking the feasibility of the given HAT is formulated as a problem for finding a solution of the following system;

$$\sum_{j:(i,j,s)\in P} x_{ijs} + \sum_{j:(j,i,s)\in P} x_{jis} = 1 \quad (\forall i \in T,\ \forall s \in S),$$

$$\sum_{s:(i,j,s)\in P} x_{ijs} + \sum_{s:(j,i,s)\in P} x_{jis} = 1 \quad (\forall i \in T,\ \forall j \in T,\ i < j),$$

$$x_{ijs} \in \{0,1\} \quad (\forall (i,j,s) \in P),$$

where T denotes the set of teams and S denotes the set of slots, i.e., $T \stackrel{\text{def.}}{=} \{1,2,\dots,2N\}$ and $S \stackrel{\text{def.}}{=} \{1,2,\dots,2N-1\}$.

By means of solving the IP, we can check feasibility of any given HAT. However, when $2N = 20$, we have to solve 8008 IP problems and it is too time consuming. We introduce a simple necessary condition for feasible HAT[2]. Assume that HA-pattern x, y, and z of the given HAT have exactly one break. If x, y and z satisfy the conditions that x has 'A' on slots $s, s+1$, y has 'A' on slots $s+1, s+2$, and z has 'A' on slots $s+2, s+3$ at a slot $s \in \{1,2,\dots,2N-4\}$, then it is not difficult to show that the given HAT is infeasible. The above constraint is called a ***triple break constraint***. By applying this constraint, the number of candidates of feasible HAT decreases drastically (Table 1). Then we solved the IP problems to check feasibility of the remained HAT.

Table 1. Number of feasible HAT

#teams	#o.&c.	#triple	#feasible
6	0	0	0
8	1	0	0
10	6	0	0
12	28	1	0
14	120	4	0
16	495	15	1
18	2002	50	0
20	8008	161	4

#o.&c.: the number of equitable HAT satisfying opening and closing conditions,
#triple: the number of the remained HAT after applying triple break constraint,
#feasible: the number of feasible HAT.

Table 1 is a result of computational experiments. Total computational time is within 3.5 hours on Windows 98 PC (CPU: Pentium II 300MHz, RAM: 128MB). The result indicates that for $2N \in \{6,8,10,12,14,18\}$, there is no feasible equitable HAT satisfying all the conditions, and few feasible HAT are found for $2N \in \{16,20\}$.

We show that there are few feasible HAT for the practical number of teams, and it takes acceptable time to enumerate all feasible HAT.

References

[1] D. de Werra: Geography, games and graphs. *Discrete App. Math.*, 2(1980), 327–337.

[2] R. Miyashiro and T. Matsui: Note on equitable round-robin tournaments. *Proceedings of 6th KOREA-JAPAN Joint Workshop on Algorithms and Computation*, 2001, 135–140.