Maximizing Interval Reliability in a Periodic Rejuvenation Model

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1. Introduction

The software rejuvenation is a proactive fault management technique for operational software systems which age due to the error conditions that accrue with time and/or load [1]. In this paper, we consider the Markov regenerative process (MRGP) model proposed by Garg *et al.* [2]. First, it is shown that the underlying MRGP model can be reduced to a simple semi-Markov (SMP) model [3]. Next, we formulate the limiting interval reliability [4] and derive the optimal rejuvenation policy maximizing it.

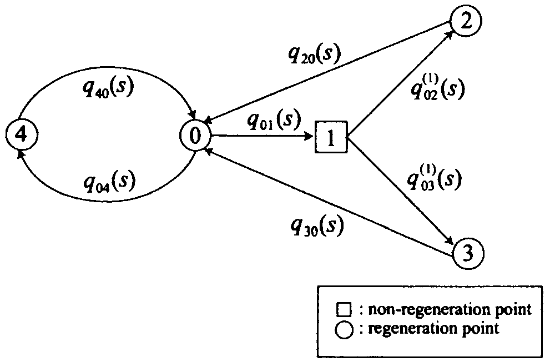

Fig. 1: Transition diagram for MRGP model.

2. Periodic Rejuvenation Model

Consider the similar MRGP model to Garg *et al.* [2] with the following five states:

State 0: highly robust state (normal operation state)

State 1: failure probable state

State 2: software rejuvenation state from failure probable state

State 3: failure state

State 4: software rejuvenation state from highly robust state.

Suppose that the software system is started for operation at time $t = 0$ and is in the highly robust state (normal operation state). Let Z_0 be the random time to reach the failure-probable state from the highly robust state. Let $\Pr\{Z_0 \le t\} = F_0(t)$ with finite mean μ_0 (> 0). Just after the state becomes the failure-probable state, a system failure may occur with a positive probability. Let X be the time to failure from the failure-probable state having the distribution function $\Pr\{X \le t\} = F_f(t)$ with finite mean λ_f (> 0). If the failure occurs before triggering a software rejuvenation, then the repair operation is started immediately. The time to complete the repair Y is also the positive random variable having the distribution function $\Pr\{Y \le t\} = F_a(t)$ with finite mean μ_a (> 0). Without any loss of generality, it is assumed that after completing repair the system becomes as good as new.

On the other hand, rejuvenation is performed at a random time interval measured from the start (or restart) of the software in the robust state. The probability distribution function of the time to invoke the software rejuvenation and the distribution function of the time to complete software rejuvenation are represented by $F_r(t)$ and $F_c(t)$ with finite means t_0 (≥ 0) and μ_c (> 0), respectively. After completing the software rejuvenation, the software system becomes as good as new, and the software age is initiated at the beginning of the next highly robust state. Figure 1 illustrates the transition diagram for the MRGP model, where the states denoted by circles (0, 2, 3, 4) and square (1) are regeneration and non-regeneration points, respectively, in the MRGP.

Garg *et al.* [2] analyze this model numerically via a Markov regenerative stochastic Petri net. However, this can be analyzed mathematically in the familiar way to the SMP analysis [3]. In fact, it is straightforward to see that the MRGP in Fig. 1 is equivalent to a SMP shown in Fig. 2, where

State 0': highly robust and failure probable state

State 1': system failure

State 2': preventive maintenance (rejuvenation).

For convenience, we pay our attention to derive the periodic rejuvenation policy, *i.e.*

$$F_r(t) = U(t - t_0) = \begin{cases} 1: & (t \ge t_0) \\ 0: & (t < t_0) \end{cases}$$

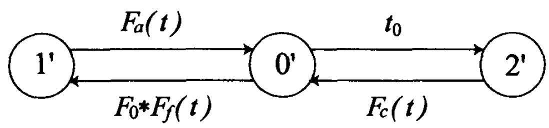

Fig. 2: Transition diagram for SMP model.

where $U(\cdot)$ is the unit step function. In the following section, we formulate the interval reliability and derive the optimal periodic rejuvenation schedule t_0^*.

3. Interval Reliability

Suppose that the rejuvenation is not performed during the time interval $[t, t+x]$. Then the interval reliability, $IR(x, t, t_0)$, is defined by

$$\begin{aligned} &IR(x,t,t_0) \\ &= \overline{F_0 * F_f}(T+x)\overline{F_r}(T) \\ &\quad + \int_0^T \overline{F_0 * F_f}(T+x-u)\overline{F_r}(T-u)dM(u), \end{aligned} \tag{1}$$

where $M(t)$ is the expected number of visits to State 0' during the period $(0, t]$ in Fig. 2 (see [3]), and in general $\overline{\phi}(\cdot) = 1 - \phi(\cdot)$. The function $IR(x, t, t_0)$ means the probability that at specified time t, the system is operating and will continue to operate for an interval of time x [4]. When $t = 0$ and $x \to \infty$, the interval reliability is reduced to the reliability function and the pointwise availability at time t, respectively.

Taking the LS transform of Eq. (1), we have

$$\begin{aligned} IR(x,s,t_0) &= \int_0^\infty e^{-st} IR(x,t,t_0)dt \\ &= \Big\{ se^{sx} \int_x^{t_0+x} e^{-st}\overline{F_0 * F_f}(t)dt \Big\} \\ &\Big/ \Big\{ 1 - F_a^*(s) \int_0^{t_0} e^{-st} d(F_0 * F_f)(t) \\ &- F_c^*(s) e^{-st_0}\overline{F_0 * F_f}(t_0) \Big\}, \end{aligned} \tag{2}$$

where $F^*(s) = \int_0^\infty e^{-st}dF(t)$. Then the limiting interval reliability, $IR_L(x, t_0)$, is defined by

$$\begin{aligned} IR_L(x,t_0) &= \lim_{t\to\infty} IR(x,t,t_0) = \lim_{s\to 0} IR(x,s,t_0) \\ &= \Big\{ \int_x^{t_0+x} \overline{F_0 * F_f}(t)dt \Big\} \Big/ \Big\{ \int_0^{t_0} \overline{F_0 * F_f}(t) \\ &+ \mu_a F_0 * F_f(t_0) + \mu_c \overline{F_0 * F_f}(t_0) \Big\} = S_L(t_0)/T_L(t_0). \end{aligned} \tag{3}$$

Thus, the problem is to seek the optimal reriodic rejuvenation policy t_0^* which maximizes the limiting interval reliability, $IR_L(x, t_0)$.

4. Optimal Rejuvenation Policy

We make the following assumption:

(A-1) $\mu_a > \mu_c$,

that is, the mean repair time is strictly larger than the mean rejuvenation overhead. Define the following non-linear function:

$$\begin{aligned} q(t_0) &= (1 - H(t_0))T_L(t_0) - \big\{1 + (\mu_a - \mu_c) \\ &\quad \times r(t_0)\big\} S_L(t_0), \end{aligned} \tag{4}$$

where

$$\begin{aligned} H(t_0) &= \frac{F_0 * F_f(t_0 + x) - F_0 * F_f(t_0)}{\overline{F_0 * F_f}(t_0)}, \\ r(t_0) &= \frac{dF_0 * F_f(t_0)/dt_0}{\overline{F_0 * F_f}(t_0)}. \end{aligned} \tag{5}$$

Theorem: (1) Suppose that the distribution function $F_0 * F_f(t)$ is strictly IFR under (A-1).

(i) If $q(\infty) < 0$, then there exists a unique optimal rejuvenation policy t_0^* $(0 < t_0^* < \infty)$ maximizing the limiting interval reliability $IR_L(x, t_0)$, and the maximum limiting interval reliability is given by

$$IR_L(x, t_0^*) = \frac{1 - H(t_0^*)}{1 + (\mu_a - \mu_c) r(t_0^*)}. \tag{6}$$

(ii) If $q(\infty) \geq 0$, then $t_0^* \to \infty$ and

$$IR_L(x, \infty) = \frac{\int_x^\infty \overline{F_0 * F_f}(t)dt}{\mu_0 + \lambda_f + \mu_a}. \tag{7}$$

(2) Suppose that the distribution function $F_0 * F_f(t)$ is DFR under (A-1). Then, the optimal rejuvenation policy becomes $t_0^* \to \infty$.

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