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Inefficiency of the Production Process

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1. Introduction

As we know, an observed or measured value of a physical phenomenon can be written as the sum of its true value and an error term, which obeys to the four conditions of Gauss-Markov [1]. But, when the error term does not obey to those conditions, we used to assume that it does and we continue our analysis, which is typically an unrealistic assumption. For this kind of systems, which present inefficiency, we propose in this paper a model of data and a technique for evaluation of the inefficiency and the different parameters of the model.

2. Concept of Production Process Inefficiency

In research and development stage sometimes we are subject to fix the values of a large number of parameters of many sub-processes. Let us suppose that we have a production line composed by 5 sub-processes and we have to fix 10 parameters at each stage. In order to evaluate the optimal values of the different parameters, based on the experience of the staff, we check the degree of master of the sub-processes and try to begin the analysis by the worst mastered one. By this assumption, it is clear that some bad settings of the controllable factors of the supposed mastered sub-processes can occur and a lack of output will be generated. To estimate the expected response of the process, an evaluation of that lack is recommended, but in order to reach the expected value, a research of the origins of the lack is necessary.

In the following classical model of data:

Observed Value = True value + Error (1)

the effect of the non-controllable factors is represented by the error term and the combination of the controllable factors is represented by the term "true value" [2, 3]. But, if we want to evaluate the lack of output (inefficiency), model (1) is not adequate. Contrarily, its use conducts to a bad estimation of the model parameters. To evaluate the inefficiency, a new term should be added to model (1). The model can be written implicitly as follows:

Observed Value = True value − Inefficiency + Disturbance (2)

where the disturbance term is the classical error of model (1).

Physically, the introduced inefficiency term represent the effects of controllable factors, which are nonrandom settings of the studied system. Therefore, conversely to the classical error, which is symmetric, extrinsic, an unbiased and a random term, we think that inefficiency is asymmetric, intrinsic, unbiased and a nonrandom term [4].

3. Formulation of the Model

Based on the previous explanations, our model can be written explicitly as follows:

$$y = f(x|\ \theta) - u(x) + v \qquad (3)$$

where y is the observed output, f is the system function, x is the input, θ is the model parameters, u is the inefficiency function ($u(x)\geq 0$), and v is the disturbance term.

For an affine system $f(x\,|\,\theta)=\theta_0+\theta_1 x$ and for a multi-step inefficiency function, model (2) can be rewritten as follows:

$$y = \theta_0 + \theta_1 x - u(x) + v \qquad (4)$$

where y is the observed output, x is the input, θ_0, θ_1 are the model parameters, v is the disturbance term, $v \sim N(0, \sigma^2)$, and

$$u(x)=\sum_{j=0}^{M}\xi_j\Delta_j \quad \text{where} \quad \xi_j=\begin{cases}1 & \text{for } T_j < x \leq T_{j+1}\\ 0 & \text{otherwise}\end{cases}$$

$T_0 = -\infty$, $T_{M+1} = \infty$ and $\Delta_0=0$. Δ_j is the value of the j^th gap, T_j is the position of the j^th gap, and M is the number of gaps.

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For searching the parameters of the previous model, the residual sum of squares should be minimized as shows the following system:

$$Se(M) = \min \sum_{j=0}^{M} \sum_{i \in I_j} \left(y_i - \hat{\theta}_0 - \hat{\theta}_1 x + \delta_j \right)^2 \quad (5)$$

where $I_j = \{ i \mid t_j < x_i \le t_{j+1} \}$, $t_0 = -\infty$, $t_{M+1} = \infty$, $\delta_0 = 0$ and $\hat{\theta}_0$, $\hat{\theta}_1$, δ_j and t_j are respectively the estimates of θ_0, θ_1, Δ_j and T_j.
It is clear that for zero gap, M=0, (5) gives the residual sum of squares of the ordinary least square (OLS) method. So, the proposed method is an extension of OLS and we call it "extended OLS".

4. Parameters Estimation and Gaps Number Search

The search of the optimal number of gaps returns to the search of the optimal number of the model parameters, which can be done through Akaike Information Criterion (AIC) [6].

The estimation of the model parameters can be done according the following steps:

Step 1: For M=0, calculate the residual some of squares $Se(0)$ according the OLS method and the corresponding AIC score, AIC(0).

Step 2: Increment M by 1, calculate the new residual sum of squares $Se(M)$ and its corresponding AIC score, AIC(M).

Step 3: compare the actual AIC score to the previously calculated score. If the actual score is larger than the previous one, then the system has (M-1) gaps. Else, go to step 2.

5. Simulation and Concluding Remarks

We check the results given by the proposed method through Monte Carlo experiment for zero, one and two gaps as reports Table 1. Based on the simulation results, it was proved that our method works for evaluation of the number and the positions of the gaps as well as the estimation of the different model parameters.

During the simulation, we noted that the computation time increase exponentially when the number of gaps increase. In fact, for a factor of L levels and for searching r gaps, we should iterate ${}_{L-1}C_r$ times, where C means combination. To decrease that time, other optimization methods will be investigated in future work.

Table 1: Results of the gaps number search.

M is the estimated number of gaps, Se is calculated according (5), p is the degree of freedom of the model, AIC(p)=NLn(Se(p))+2p, where N is the number of samples. t_1, δ_1, t_2, δ_2, t_3 and δ_3 are respectively the estimates of the true position and values of the gaps T_1, Δ_1, T_2, Δ_2, T_3 and Δ_3.

	no gaps		one gap			two gaps			
	gap=0		Δ_1=5, T_1=10			Δ_1=10, T_1=7, Δ_2=20, T_2=14			
M	0	1	0	1	2	0	1	2	3
Se	20.6	17.9	46.7	10.9	9.51	168.49	126.56	21.29	17.71
p	1	3	1	3	5	1	3	5	7
AIC	62.2	63.7	78.9	53.8	55.1	104.54	102.81	71.16	71.48
t_1		19		10	10		7	7	2
δ_1		-1.68		5.37	5.25		5.4	10.6	-1.81
t_2					14			14	7
δ_2					4.28			20.25	8.44
t_3									14
δ_3									17.29
$\hat{\theta}_0$	29.9	30.1	32.3	30.8	31.2	34.49	34.12	30.26	29.46
$\hat{\theta}_1$	5.01	4.98	4.57	4.97	4.90	3.63	4.00	4.96	4.84

The usefulness and the worth of the proposed method were checked through a case study of development of a new resin, which will be exposed in details during the presentation.

References

[1] Bilal M. Ayyub, Richard H. McCuen, Probability, statistics, & reliability for engineers, Boca Raton: Press, 1997.

[2] Dick R. Wittink, The application of regression analysis, Allyn and Bacon, Inc., 1988.

[3] P. McCullagh, J. A. Nelder, Generalized linear models, Chapman and Hall, 1989.

[4] Ilyes Bellamine, Hiroshi Morita, Evaluation of the single step inefficiency function, European Journal of Operational Research, submitted.

[5] Tim Coelli, D. S. Prasada Rao and George E. Battese, An introduction to efficiency and productivity analysis, Kluwer Academic Publishers, 1998.

[6] Y. Sakamoto, M. Ishiguro, and G. Kitagawa, Akaike information criterion statistics, Tokyo: KTK Scientific Publishers, Dordrecht, Tokyo: D. Reidel, 1986.

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