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The Method of Elastic Constraints for Multiobjective Combinatorial Optimization and its Application in Airline Crew Scheduling Problems

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Multiobjective combinatorial optimization (MOCO) problems are in general very hard to solve because they are usually $\mathcal{NP}$-hard, $\#\mathcal{P}$-hard, and an exponential number of efficient solutions may exist.

Solution techniques make use of some scalarization that transforms the multiobjective problem into a single objective one. In order to find all efficient solutions of the MOCO problem, the scalarized problem has to be solved repeatedly, for different choices of the parameters involved. Scalarization techniques include the use of weights for aggregating objectives and/or constraints on values of objective functions.

It is well known that the most popular scalarization technique of minimizing the weighted sum of objectives cannot generate all efficient solutions in the MOCO case. In this paper we will first review some other methods: the ε-constraint method, the augmented weighted Tchebycheff method, and Benson's method. These methods are known to generate the complete efficient set even for non-convex multicriteria optimization problems, including MOCO. A scalarization of a MOCO problem is required to be a linear integer programming problem. This implies that the scalarized problems used in all these methods have to be formulated in a way that involves constraints on the value of some objectives. These constraints often imply that the scalarized problem is $\mathcal{NP}$-hard and they lead to considerable computational difficulties when solving the scalarized problems on a computer, because they destroy the structure that is exploited by integer programming codes.

We also show that Lagrangean relaxation applied to these problems always leads to weighted sum problems, so that subgradient methods do not resolve the dilemma we face when trying to solve MOCO problems by scalarization.

We propose a new scalarization technique. The MOCO problem

$$\min\left\{(c_1^t x, \ldots, c_Q^t x) : x \in X\right\}, \tag{1}$$

where X is some finite feasible set is scalarized as follows:

$$\begin{aligned} \min_{x \in X} c_i^t x + \sum_{j \neq i} p_j s_{p_j} & & \\ \text{subject to } c_j^t x + s_{l_j} - s_{p_j} &= \varepsilon_j \quad j \neq i. \end{aligned} \tag{2}$$

In this formulation, the feasible set in terms of variables x is exactly the same as before, i.e. X: The constraints on objective values are "elastic", because $c_j^t x$ can be less than, equal to or greater than ε_j. However, we penalize values greater than ε_j

The main results concerning the method are the following theorems.

Theorem 1 *An optimal solution of (2) is efficient for (1), if $p_j > 0$ for all $j \neq i$.*

Theorem 2 *Let x^* be an efficient solution of (1). Then there is some $\varepsilon \in \mathbb{R}^Q$ and for each $i = 1, \ldots, Q$ a vector $p^i \in \mathbb{R}^{Q-1}$ such that x^* defines an optimal solution of (2) for this i for all penalty vectors $p \in \mathbb{R}^Q$ with $p_j \geq p_j^i, j \neq i$.*

We also show that the method of elastic constraints can be seen as a common generalization of the weighted sum technique and the ε-constraint method. We illustrate the results using a small bicriteria linear integer programming problem.

Finally, we report our experience of applying the method to solve bicriteria crew scheduling problems arising in airline operations. The objectives are to minimize cost and to maximize robustness, an objective function the purpose of which is to reduce the effects of delays. The problem can be formulated as bicriteria generalized set partitioning problem as follows:

$$\begin{aligned} \min z^1 &= c^T x \\ \min z^2 &= r^T x \\ \text{subject to } A_1 x &= e \\ A_2 x &= b \\ x &\in \{0,1\}. \end{aligned} \tag{2SPP}$$

With the method of elastic constraints we were able to solve this problem. This was not possible with the ε-constraint method because of unacceptable computation times, and the weighted sums method, which only found few efficient solutions. To the authors' knowledge these are the largest MOCO problems solved successfully to date.