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An Analysis of Competitive Diffusion Model and Its Application to Korean Mobile Phone Service Market

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ABSTRCT

Diffusion models were developed to explain the diffusion process of new product introductions in a market and utilized to forecast the demand of new products or services. Particularly, as the information and communication industry is becoming important in recent, the diffusion models are paid attention to as an effective tool for the demand analysis of subscription of new telecommunication services such as mobile phone, high speed internet service, and so forth. The objective of this research is the development of new diffusion model which is explicitly considering competitive market situation and its empirical application to the demand analysis of mobile phone service market. In this paper, a competitive diffusion model, which is the extension of ordinary Bass model by incorporating competitive situation, is suggested. And the existence conditions for explicit solutions of the model are derived. Furthermore equilibrium points and its stability conditions are also characterized. Finally in order to show the applicability of the model, we estimate the demand function of mobile phone subscription in Korea and the explanatory power of the suggested diffusion model is examined by comparing with the monopolistic diffusion model.

1. Introduction

Telecommunication industry plays an important role as a technological infrastructure in the transition process to advanced information society. And it's market environments have experienced drastic changes to respond to various social as well as economic needs. Among the changes of telecommunication market environments, the most influential and noteworthy one is that its market structure undergoes a shift from natural monopoly to competitive market.

In 1990, the Korean regulator manifested the introduction of competition in telecommunication industry in order to provide high-quality and low-price service to people by inducing competition among providers and to enforce domestic providers' competitive advantage to cope with market open to foreign companies. A series of appropriate policies of restructuring telecommunication industry has been carried out. As a result, starting from the international phone market in 1991, the restructuring policy continues until now, even in the market of local phone service. Now, the Korean telecommunication market consists of 12 business areas where 35 providers are competing.

Diffusion models were developed to explain the

growth of cells or species in ecology. Since the seminal work of Bass(1969), marketing researches adopted diffusion models in the explanation of the diffusion process of new product introductions in a market and utilized to forecast the demand of new products or services. Particularly, as the information and communication industry is becoming important in recent, the diffusion models are paid attention to as an effective tool for the demand analysis of subscription of new telecommunication services such as mobile phone, high speed internet service, and so forth.

It should be noted that most previous diffusion models including Bass(1969) assumed product market as monopoly. So there remains a paucity of diffusion model literature that considers competition mechanism in diffusion process. However if the good we want to study is traded in competitive market, its demand characteristics are apparently different from the good in monopolistic market. And monopoly based diffusion model is insufficient to capture and explain the diffusion characteristics of the good in competitive market. Specifically, in the demand analysis of telecommunication market using diffusion model, as the real market shows aforementioned structural change, from natural monopoly to competitive market, it is necessary to develop a model which incorporates competition explicitly.

The objective of this research is the development of new diffusion model which is explicitly considering competitive market situation and its empirical application to the demand analysis of mobile phone service market. In this paper, a competitive diffusion model, which is the extension of ordinary Bass model by incorporating competitive situation, is suggested. And the existence conditions for explicit solutions of the model are derived. Furthermore equilibrium points and its stability conditions are also characterized. Finally in order to show the applicability of the model, we estimate the demand function of mobile phone subscription in Korea and the explanatory power of the suggested diffusion model is examined by comparing with the monopolistic diffusion model.

2. Model analysis

The development of diffusion models in marketing is based on the behavioral rationale that a new product is first adopted by a certain group of buyers, called innovators, who then influence others by word-of-mouth to adopt the product. The well-known Bass model (1969) captures this dynamic diffusion mechanism as the following simple differential equation:

$$\frac{dN}{dt} = (a + bN)(m - N) \qquad (1)$$

where N is the cumulative number of product adopters, m is the total size of potential market demand, a is the coefficient of innovation, and $b = b'/m$, where b' is the coefficient of imitation. The above equation represents that the rate of change in the cumulative number of adopters is proportional to a linear function of the cumulative adopters until then and the remaining market potential.

We now extend the Bass model by considering diffusions of two products in a market competing each other for the same potential demanders. The demand of each product is influenced not only by the diffusion of its own product but also by that of the other. Denoting by N_i the cumulative number of adopters of product i (for $i=1,2$), it is reasonable to suppose that

$$\frac{dN_1}{dt} = (m - N_1 - N_2)(a_1 + b_{11}N_2 + b_{12}N_2) \quad (2)$$

$$\frac{dN_2}{dt} = (m - N_1 - N_2)(a_1 + b_{21}N_2 + b_{22}N_2) \quad (3)$$

Here a_i and b_{ii} are the Bass model's coefficient for product i if its market is monopoly. Also b_{ij} measures degree to which the presence of product j affects the diffusion of product i. We call this model the competitive Bass model.

In general, the simultaneous differential equations (2) and (3) can not be explicitly solved. We consider a particular set of conditions in which they can be solved. In analyzing the growth of populations of certain species in ecology, the model which considers populations of twp species living together and competing each other for the same limiting resource is called Lotka-Volterra equation. The existence conditions discussed here is in line with the those of Lotka-Volterra equation suggested by Pielou (1969)

The simplification condition that allows solution of the competitive Bass model consists in assuming that for an individual adopter of either product the influencing effect of all other adopters of both products is the same. This assumption can be described mathematically as follows.

$$N = N_1 + pN_2 \quad (4)$$

Making this condition, Eqs. (2) and (3) may be replaced by

$$\frac{dN_1}{dt} = (m - N_1 - N_2)(a_1 + b_{11}N_1 + b_{12}N_2) = (m - N_1 - N_2)(a_1 + b_1N) \quad (5)$$

$$\frac{dN_2}{dt} = (m - N_1 - N_2)(a_1 + b_{21}N_2 + b_{22}N_2) = (m - N_1 - N_2)(a_2 + b_2N) \quad (6)$$

It then follows that

$$b_{11}N_1 + b_{12}N_2 = b_1N = b_1(N_1 + pN_2)$$
$$b_{21}N_1 + b_{22}N_2 = b_2N = b_2(N_1 + pN_2)$$

Then we can get

$$b_{11} = b_1, \quad \frac{b_{12}}{b_{11}} = p$$

$$b_{21} = b_2, \quad \frac{b_{22}}{b_{21}} = p$$

or $\quad b_{12}b_{21} = b_{11}b_{22}$

This relation is therefore equivalent to the assumption already made. And by summing up Eqs. (5) and (6), we can get

$$\frac{dN_1}{dt} + \frac{dN_2}{dt} = (m - N_1 - N_2)(a_1 + a_2 + b_1N + b_2N) \quad (7)$$

$$\frac{dN_1}{dt}\frac{1}{(m-N_1-N_2)}+\frac{dN_2}{dt}\frac{1}{(m-N_1-N_2)} \quad (8)$$
$$=[a_1+a_2+(b_1+b_2)N]$$

Eqs. (5) and (6) can now be rewritten

$$\frac{d}{dt}\ln(m-N_1-N_2) = -\frac{dN_1}{dt}\frac{1}{(m-N_1-N_2)} = -(a_1+b_1N) \quad (9)$$

$$\frac{d}{dt}\ln(m-N_1-N_2) = -\frac{dN_2}{dt}\frac{1}{(m-N_1-N_2)} = -(a_2+b_2N) \quad (10)$$

Multiplying b_2 by Eq. (9) and b_1 by Eq. (10) gives

$$\frac{d}{dt}[b_2\ln(m-N_1-N_2)] = -a_1b_2-b_1b_2N \quad (11)$$

$$\frac{d}{dt}[b_1\ln(m-N_1-N_2)] = -a_2b_1-b_1b_2N \quad (12)$$

By subtracting Eq. (12) from Eq. (11), we can eliminate N as follows.

$$\frac{d}{dt}[b_2\ln(m-N_1-N_2)-b_1\ln(m-N_1-N_2)] = a_2b_1-b_1b_2 \quad (13)$$

Eq. (13) can be written as

$$\frac{d}{dt}\ln[\frac{(m-N_1-N_2)^{b_2}}{(m-N_1-N_2)^{b_1}}] = -a_2b_1-a_1b_2 \quad (14)$$

$$\frac{d}{dt}\ln[(m-N_1-N_2)^{b_2-b_1}] = a_2b_1-a_1b_2 \quad (15)$$

The final solution of competitive Bass model can be derived as follow.

$$(m-N_1-N_2)^{b_2-b_1} = C\cdot\exp[(a_2b_1-a_1b_2)t] \quad (16)$$

3. Conclusion and further studies

In this paper, we proposed competitive Bass model and attempt mathematical analysis on the equation. Particularly a condition with which the competitive Bass model possesses explicit form solution is suggested and proved.

As further works, equilibrium points and its stability conditions of the model will be characterized and in order to show the applicability of the model, we will estimate the demand function of mobile phone subscription in Korea and the explanatory power of the suggested diffusion model will be examined by comparing with the monopolistic diffusion model.

Reference

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[2] Pielou, E. C.: *An introduction to mathematical ecology*, Wiley Interscience, New York, NY, (1969)