Concept of the Systematic Inefficiency and its Applications

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Introduction

The concept of efficiency has been introduced in economics as the ability of a firm to operate with the minimum level of resources necessary to produce its output and yet remain highly competitive over the time. In this context, the inefficiency can be defined as the use of excessive input to produce its output or the under production of output given its input. To estimate efficiency two families of methods have been developed: parametric and non-parametric methods. The former methods involve the econometric estimation of parametric functions, while the latter do not. One of the main parametric methods is the stochastic frontier production function, in which the error term, ε, is defined as the sum of a disturbance term, v_i, and a non-negative stochastic variable u_i. It was assumed that the v_is were independent and identically distributed (i.i.d.) normal random variables with mean zero and constant variance, independent of the u_is, which were assumed to be i.i.d exponential, truncated normal or two-parameter gamma stochastic variables. The main criticism of this method is that: different distributions for the stochastic inefficiency u lead to different estimates for the production function parameters, and there are no good a priori arguments for any particular distributional form. On the other hand, one of the main non-parametric methods is the data envelopment analysis, which is based on the maximization of the ratio: weighted sum of output vs. weighted sum of input. This model does not use any functional form of the production function. But, it is very sensitive to the noise of the data. The both families of methods are used to benchmark firms in order to improve their performances.
Having been inspired from the stochastic frontier production model we introduce in this paper the concept of the estimation of the inefficiency by a non-stochastic (multi-step) function. Following this introduction, the concept is developed and two applications are proposed.

The Concept

In quality control domain, it has been supposed that the output of a physical phenomenon, y, is a sum of a systematic term, $f(x)$, and a disturbance term, v, as follows

$$y=f(x)+v \tag{1}$$

However, if for some levels (or settings) the intrinsic properties of the input change and provoke a lack of output, model (1) does not work. Let's suppose that the physical state of a chemical substance changed from the setting x=10 giving the output plotted in Figure 1.

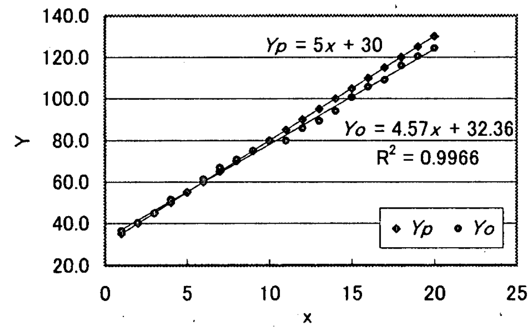

Figure 1: Plot of the perfect output *Yp* and the observed output *Yo*.

Based on the scatter-plot of the observed output and the high value of the correlation coefficient R^2=0.9966, we can conclude that the obtained output is acceptable and can be modeled as Yo=4.57x+32.36. But, really the perfect output is Yp=5x+30, and due to the change of some intrinsic properties of the input, the observed output was deviated from the perfect one, hence the limit of the model (1) for detecting this lack of output.
In this paper, we propose to decompose the systematic term into two terms one representing the best output obtained from the mix of the used input noted $g(x)$ and the other a positive term noted $u(x)$ representing a lack of output caused by the change of some intrinsic properties of the input and/or a change of settings of some supposed fairly stable (or mastered) factors that are confused into the disturbance term. The proposed model is defined by

$$y = g(x) - u(x) + v \tag{2}$$

The term $u(x)$ enables to assess the difference of performance between the best observations and the others. Therefore we call it inefficiency. In this context, the term "perfect" means the optimum output *obtainable* from the input and the term "best" means the best output *obtained* from the used input, which is not necessarily the optimum.

In the stochastic frontier model, the inefficiency is assumed to be a component of the error term. It means that *u(x)* is a random factor, such as the will and the effort of the producer and his employees, and perhaps such factors as defective and damaged product. However, in order to estimate the inefficiency related to the intrinsic properties of the input such as the change of some physical characteristics of the components, we suggested accounting the inefficiency as a component of the systematic term and we propose to estimate it by a multi-step function. Because, we think that any function can be estimated by a sum of elementary step functions. The intrinsic properties of the input are non-stochastic factors. Therefore it is more reasonable to compute their effects by a systematic term.

The proposed concept enables to decompose the levels of input into zones of different performances and helps analysts to diagnose the causes of inefficiency of their systems.

Application 1: Development of a new resin

The proposed concept can be used in many fields such as quality control, social science, economics, etc. In this section we propose to study the problem of mixing ingredients, which is a typical application in the quality control domain. To develop a new resin usually we are subject to set up the values of dozens of parameters. After a careful study of the resin manufacturing process, we noticed that for many mixtures the variation in the resin mixture proportions is important whereas the variations of the settings of the kneading, grinding and injection parameters values are not too remarkable. In fact, according to the mixture only a fine tuning of these parameters should be done. At first, we assume that the parameters of the kneading, grinding and injection sub-processes are mastered and we select the proportions of the ingredients as key factors. However, if for some mixtures the intrinsic characteristics of some component change or bad settings of the supposed mastered parameters occur, a lack of output will be generated. This lack is called inefficiency. In order to evaluate its value and determine the ranges of input where the system performs better, we propose to use the proposed model.

Application 2: Control of a fish population

Suppose that a breeder want to cultivate some kind of fishes. We assume that the fish population is controlled only by the quantities of oxygen and nutrients. The other factors such as the velocity of the wind, the number of cloudy days, the amount of radiation, some unknown factors, etc. can be supposed as fairly constant and can be included into the disturbance term. However, if the intrinsic properties of the nutrients change or if one or more of the supposed constant factors change, then surely the system's performance will change. For example, if the calcium content of nutrients changes, the breeding process will be decelerated or accelerated and therefore the overall product of the farm will respectively decrease or increase. In order to assess the variation of performance and diagnose the possible causes, we propose to implement the proposed model and estimate the inefficiency by a multi-step function.

The proposed new model enables to decompose the levels of the retained factors into different zones where the performance of the system is different. The zones of good performance even though they are an indication of an exceptional improvement in productivity, are cause for investigation, since they may benefit farmer to find the reason and adopt practices, which lead to amelioration the yield of their processes.

Other examples and some numerical results will be shown during in the presentation.

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