Real Options in an Oligopoly Market

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1 Introduction

The aim of this paper is to analyze the strategies of firms in an oligopoly market when the underlying state variable follows a geometric Brownian motion. It is shown that, even in the oligopoly case, three types of equilibria exist as in the duopoly case. The presence of strategic interactions may push the firms to invest earlier than the optimal trigger point.

2 The Model

Consider $n \in \mathrm{N}$ firms in a market with the opportunity to make an irreversible investment. All firms can be active on the market to produce a single product and compete with each other to maximize their profits. The revenue of each firm is uncertain depending on the future condition of the market. We describe this uncertainty by state variable $X = (X_t)_{t \in \mathrm{R}_+}$ which evolves in time according to:

$$\frac{\mathrm{d}X_t}{X_t} = \mu \mathrm{d}t + \sigma \mathrm{d}z_t, \qquad t \geq 0, \tag{1}$$

where $(z_t)_{t \in \mathrm{R}_+}$ denotes the standard Brownian motion, and where μ and σ is positive constants. As usual, the mean growth rate is assumed to be strictly less than the risk-neutral discount factor r, i.e. $\mu < r$. Also, it is assumed throughout that the firms are risk neutral.

Suppose that k, $k \leq n$, firms are active on the market. The revenue flow of each active firm is given by $\rho(x) = \pi_k x$, $\pi_k \in \mathrm{R}_+$, when $X_t = x$. The constants π_k are non-increasing in k, $k = 1, 2, \ldots, n$, meaning that an investment is less profitable when more firms have invested. This situation is called a strategic substitution and often observed in a market consisting of competing firms. The sunk cost for each firm to adopt the investment opportunity is assumed to be a constant and equal to $I \in \mathrm{R}_+$. That is, all firms on the market have the same technology (symmetric firms). Each firm seeks an optimal adoption strategy to the investment opportunity.

3 Sequential Investments

Let τ_j denote the adoption time of firm j, and let x_j be the associated trigger point , i.e., $\tau_j = \inf\{t \in \mathrm{R}_+ : X_t = x_j\}$. Associated with the collection of the adoption (stopping) times are the ordered sequence of random times $\tau_{(1)} < \tau_{(2)} < \cdots < \tau_{(n)}$. That is, by definition, $\tau_{(1)} = \min\{\tau_1, \tau_2, \ldots, \tau_n\}$, and

$$\tau_{(k+1)} = \min\{\tau_j : j \in \{1, \ldots, n\},\ \tau_j > \tau_{(k)}\},$$

$k = 1, 2, \ldots, n-1$. In particular, $\tau_{(n)} = \max\{\tau_1, \tau_2, \ldots, \tau_n\}$. Similarly, we denote the ordered sequence of the thresholds by $x_{(k)}$, $k = 1, 2, \ldots, n$.

Suppose that $\{\tau_{(k-1)} < t\}$, i.e. there are k firms that have not invested before time t. In this oligopoly situation, the value function of the leader among the k firms is given by

$$\begin{aligned} V_{(k)}(x) \;=\; \operatorname*{ess\,sup}_{\tau^*_{(k)} \in \mathcal{T}_t} \Bigg\{ & \mathrm{E}^x \left[\int_{\tau^*_{(k)}}^{\tau^*_{(k+1)}} \mathrm{e}^{-r(s-t)} \pi_k X_s \mathrm{d}s \,\middle|\, \mathcal{F}_t \right] \\ & - \mathrm{E}^x \left[\mathrm{e}^{-r(\tau^*_{(k)}-t)} I \,\middle|\, \mathcal{F}_t \right] \\ & + \mathrm{E}^x \left[\sum_{j=k+1}^{n-1} \int_{\tau^*_{(j)}}^{\tau^*_{(j+1)}} \mathrm{e}^{-r(s-t)} \pi_j X_s \mathrm{d}s \,\middle|\, \mathcal{F}_t \right] \\ & + \mathrm{E}^x \left[\int_{\tau^*_{(n)}}^{\infty} \mathrm{e}^{-r(s-t)} \pi_n X_s \mathrm{d}s \,\middle|\, \mathcal{F}_t \right] \Bigg\} \end{aligned}$$

where $\tau^*_{(j)}$ denotes the optimal adoption time for the j-th follower obtained above with $\tau^*_{(n+1)} = \infty$.

Proposition 3.1 The optimal threshold $x^*_{(k)}$ to adopt the investment opportunity is given by

$$x^*_{(k)} = \frac{\beta}{\beta-1}\frac{r-\mu}{\pi_k}I. \qquad (2)$$

The trigger point $x^*_{(k)}$ is strictly decreasing in k and increasing in volatility σ.

4 Equilibria

Suppose that $(k-1)$ firms have already invested, and suppose that firm i, among the remaining $(n-k+1)$ firms, decides to invest immediately. Then, the value function for firm i (the leader) is given by

$$V^m_{(k)}(x) = \frac{\pi_k}{r-\mu}x - I - \sum_{j=k+1}^{n}\left(\frac{x}{x^*_{(j)}}\right)^{\beta}\frac{\pi_{j-1}-\pi_j}{r-\mu}x^*_{(j)},$$

where $x < x^*_{(k+1)}$. The value function $V^m_{(k)}(x)$ represents the present value of the profit obtained after carring out the investment, and satisfies the value-matching condition $V^m_{(k)}(x^*_{(k+1)}) = V^m_{(k+1)}(x^*_{(k+1)})$.

On the other hand, suppose that firm i decides to be a follower and take the $(k+1)^{\text{th}}$ investment opportunity. The value function for firm i is given by

$$V^o_{(k+1)}(x) = \left(\frac{x}{x^*_{(k+1)}}\right)^{\beta}\frac{I}{\beta-1} - \sum_{j=k+2}^{n}\left(\frac{x}{x^*_{(j)}}\right)^{\beta}\frac{\pi_{j-1}-\pi_j}{r-\mu}x^*_{(j)},$$

where $x < x^*_{(k+1)}$. The value function $V^o_{(k+1)}(x)$ represents the option value to delay the investment opportunity, and satisfies not only the value-matching condition $V^o_{(k+1)}(x^*_{(k+1)}) = V^m_{(k+1)}(x^*_{(k+1)})$, but also the smooth-pasting condition $V^{o\prime}_{(k+1)}(x^*_{(k+1)}) = V^{m\prime}_{(k+1)}(x^*_{(k+1)})$.

Lemma 4.1 For all $k = 1, 2, \ldots, n-1$, we have:
(1) The value function $V^m_{(k)}(x)$ is strictly concave in $x \in (0, x^*_{(k+1)})$, while the value function $V^o_{(k+1)}(x)$ is strictly convex in $x \in (0, x^*_{(k+1)})$.
(2) We have $V^m_{(k)}(x) > V^m_{(k+1)}(x)$ and $V^o_{(k)}(x) > V^o_{(k+1)}(x)$ for $x \le x^*_{(k)}$.

Since $V^m_{(k)}(x)$ is strictly concave and $V^o_{(k+1)}(x)$ is strictly convex in $x \in (0, x^*_{(k+1)})$, there exists a unique root $x^{\mathrm{P}}_{(k)} \in (0, x^*_{(k+1)})$ for the equation $V^m_{(k)}(x) = V^o_{(k+1)}(x)$ (see Figure 1).

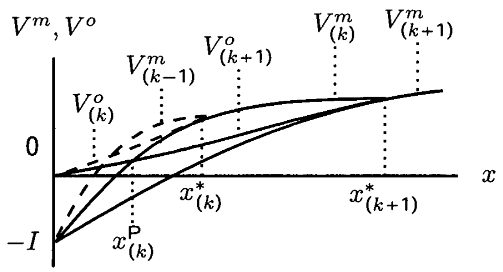

Figure 1: Value functions for the oligopoly market

The next result is a direct consequence of the properties of the value functions stated above. We are now in a position to state our main result.

Theorem 4.1 Suppose that there are n firms that have not invested in the market. Then, we have the following outcomes for the initial state x.
If $x^{\mathrm{P}}_{(k)} \le x < x^{\mathrm{P}}_{(k+1)}$, then either
(k-1) some k firms invest now, the other $(n-k)$ firms invest at $\tau^{\mathrm{P}}_{(k+1)}, \cdots, \tau^{\mathrm{P}}_{(n-1)}, \tau^*_{(n)}$ in sequence,
(k-2) some m, $m > k$, firms invest simultaneously now, the other $(n-m)$ firms invest simultaneously or sequentially after time $\tau^{\mathrm{P}}_{(m+1)}$, or
(k-3) all firms invest simultaneously now.

From Theorem 4.1, we note that there is the possibility of sequential equlibria in our oligopoly market. Note also that, in the interval $\{x^{\mathrm{P}}_{(k)} \le x < x^{\mathrm{P}}_{(k+1)}\}$, although the outcome that only one of the firms invests is a Pareto optimum, there is the possibility that more than two firms invest simultaneously.

5 Conclusions

We demonstrate that, even in the oligopoly case, three type of equilibria exist as in the duopoly case. The presence of strategic interactions may push the firms to invest earlier than the optimal trigger point.