Stock Repurchase Policy with Transaction Costs under Jump Risks

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Introduction

Stock repurchase has become an important method of distributing cash flows to shareholders. Firms repurchase stock for the following reasons: to distribute excess cash flow; to announce that firms' managers think their firms' stocks are undervalued; to avoid unwanted takeover attempts; and, to counter the dilution effects of employee and management stock options.

We assume that a firm's accumulated net revenues is governed by a jump-diffusion process, and that the firm distributes cash flows to shareholders in the form of dividends or stock repurchases. We assume that dividends represent stable cash distribution, so that the firm constantly pays out dividends by the same amount at each dividend time. On the other hand, we assume that stock repurchases represent temporary cash distribution. Thus, we concentrate how the firm repurchases stocks and examine an optimal stock repurchase policy. In this context, we assume that when the firm repurchases stock, it incurs both fixed and proportional transaction costs.

The Model

We assume that a firm's accumulated cash reserves X_t jumps at the random time $\mathcal{T}_1, \cdots, \mathcal{T}_n, \cdots$ and that the relative change in its value at a jump time is given $Y_1, \cdots, Y_n, \cdots$ respectively. We assume that $\mathcal{T}_n$ are the jump times of a Poisson process $N := (N_t)_{t\geq 0}$ with intensity $\lambda \in (0,1)$ and a sequence $Y = (Y_n)_{n\geq 1}$ is independent, identically distributed random variables taking values in $(-\infty, \infty)$ with distribution F. Assume that Y has a finite mean. Between jumps the process of the firm's accumulated cash reserves $X := (X_t)_{t\geq 0}$ follows a Brownian motion with drift. Assume that a filtered probability space $(\Omega, \mathcal{F}, \mathbb{P}; (\mathcal{F}_t)_{t\geq 0})$ satisfying the usual conditions is given. The filtration $(\mathcal{F}_t)_{t\geq 0}$ is generated by a Brownian motion process $W := (W_t)_{t\geq 0}$, a Poisson process N and the sequence Y. We assume that W, N and Y are mutually independent.

Let ζ_i be amount of the ith amount of stock repurchased. Let τ_i be the ith stock repurchase time. A stock repurchase policy is defined as the following double sequences: $v := \{(\tau_i, \zeta_i)\}_{i\geq 0}$. If the stock repurchase policy v is given, then the cash reserve of the firm, $X^{x,v} := (X_t^{x,v})_{t\geq 0}$ is given by

$$\begin{cases} dX_t^{x,v} = \mu dt + \sigma dW_t + X_{t-}Y_t dN_t, & \tau_i \leq t < \tau_{i+1} \leq T, \quad i \geq 0; \\ X_{\tau_i}^{x,v} = X_{\tau_i -}^{x,v} - \zeta_i; & X_{0-}^{x,v} = x, \end{cases} \tag{1}$$

where $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}\backslash\{0\}$ are constants. T represents a bankruptcy time. Let $\mathcal{V}$ denote the set of admissible stock repurchase policies.

The expected total discounted stock repurchases function associated with the stock repurchase policy v is defined by

$$J(x;v) = \mathbb{E}\left[\sum_{i=1}^{\infty} e^{-r\tau_i} K(\zeta_i) 1_{\{\tau_i < T\}}\right], \tag{2}$$

where $r \in \mathbb{R}_{++}$ is a discount factor. K is the stock repurchases function defined by $K(\zeta) := k_1\zeta - k_0$, where $(1-k_1) \in (0,1)$ is the proportional parameter of transaction cost and $k_0 \in \mathbb{R}_{++}$ is the fixed transaction cost. We assume that $J(0;v) = 0$. Therefore the firm's problem is to maximize eq. (2) over $v \in \mathcal{V}$:

$$V(x) = \sup_{v\in\mathcal{V}} J(x;v) = J(x;v^*). \tag{3}$$

Analysis

In this section, we prove that a stock repurchase policy for the problem (3) is optimal. To this end, we introduce the following definitions:

Definition 1 (QVI). *Let ϕ be a stochastically C^2 function. The following relations are called the QVI for the problem (3):*

$$\mathcal{L}\phi(x) \leq 0, \quad \text{for a.a. } x \text{ w.r.t. } \mathbb{G}(\cdot, x; v),\ \forall v \in \mathcal{V}; \tag{4}$$

$$\phi(x) \geq \mathcal{M}\phi(x); \tag{5}$$

$$[\mathcal{L}\phi(x)][\phi(x) - \mathcal{M}\phi(x)] = 0, \quad \text{for a.a. } x \text{ w.r.t. } \mathbb{G}(\cdot, x; v),\ \forall v \in \mathcal{V}, \tag{6}$$

where $\mathcal{L}$ is the integrodifferential operator and $\mathcal{M}$ denote the stock repurchase operator, and $\mathbb{G}$ is the Green measure.

Definition 2 (QVI policy). *Let ϕ be a solution of the QVI. Then, the following stock repurchase policy $\hat{v} = \{(\hat{\tau}_i, \hat{\zeta}_i)\}_{i \geq 0}$ is called a QVI policy:*

$$(\hat{\tau}_0, \hat{\zeta}_0) = (0, 0); \tag{7}$$

$$\hat{\tau}_i = \inf\{t \geq \hat{\tau}_{i-1}; X_t^{x,\hat{v}} \notin H\}; \tag{8}$$

$$\hat{\zeta}_i = \arg\max\{\phi(\eta(X_{\hat{\tau}_i -}^{x,\hat{v}}, \zeta_i)) + K(\zeta_i); \zeta_i\} \text{ if } \hat{\tau}_i < T. \tag{9}$$

In this context, H is the continuation region defined by $H := \{x; \phi(x) > \mathcal{M}\phi(x)\}$ and $X_t^{x,\hat{v}}$ is the result of applying the stock repurchase policy $\hat{v} = \{(\hat{\tau}_i, \hat{\zeta}_i)\}_{i \geq 0}$.

Now we can prove that a QVI policy is optimal.

Theorem 1. *A function satisfies the QVI is the value function. Furthermore, the QVI policy is optimal.*

Approximation

In this section, we verify the function satisfies the QVI by using the approximation method.

Let $\mathcal{V}_i$ consists of all admissible stock repurchase policies v_i with at most i repurchases. Here $v_i = (\tau_1, \tau_2, \cdots, \tau_i, \tau_{i+1}; \zeta_1, \zeta_2, \cdots, \zeta_i)$ with $\tau_{i+1} = \infty$ a.s. From the firm's problem eq. (3), we define the firm's problem with at most i times repurchases:

$$V_i(x) = \sup_{v_i \in \mathcal{V}_i} J(x; v_i), \quad i = 0, 1, 2, \cdots. \tag{10}$$

Put $\phi_0(x) = 0$. Let define ϕ_i for $i \geq 1$ as a solution to

$$\mathcal{L}\phi_i(x) \leq 0, \quad \text{for a.a. } x \text{ w.r.t. } G(\cdot, x; v_i),\ \forall v_i \in \mathcal{V}_i; \tag{11}$$

$$\phi_i(x) \geq \mathcal{M}\phi_{i-1}(x); \tag{12}$$

$$[\mathcal{L}\phi_i(x)][\phi_i(x) - \mathcal{M}\phi_{i-1}(x)] = 0, \quad \text{for a.a. } x \text{ w.r.t. } G(\cdot, x; v_i),\ \forall v_i \in \mathcal{V}_i. \tag{13}$$

By Definition 2 we define QVI policy with at most i times repurchases, $\hat{v}_i = \{(\hat{\tau}_j, \hat{\zeta}_j)\}_{1 \leq j \leq i}$:

$$\hat{\tau}_j = \inf\{t \geq \hat{\tau}_{j-1}; X_t^{x,\hat{v}_i} \notin H_j\}; \tag{14}$$

$$\hat{\zeta}_j = \arg\max\{\phi(\eta(X_{\hat{\tau}_j -}^{x,\hat{v}_i}, \zeta_i)) + K(\zeta_j); \zeta_j\} \text{ if } \hat{\tau}_j < T. \tag{15}$$

In this context, H_j is defined by $H_j := \{X_t^{x,\hat{v}_i}; \phi_{i-j}(X_t^{x,\hat{v}_i}) > \mathcal{M}\phi_{i-j-1}(X_t^{x,\hat{v}_i})\}$ and $X_t^{x,\hat{v}_i}$ is the result of applying the stock repurchase policy $\hat{v}_i = \{(\hat{\tau}_j, \hat{\zeta}_j)\}_{1 \leq j \leq i}$. Then, we have the following results:

Theorem 2. *The sequence $\phi_i(x)$ satisfies eq. (10). Furthermore, suppose that $\hat{v}_i \in \mathcal{V}_i$, then $\hat{v}_i$ is optimal.*

Theorem 3.

$$\lim_{i \to \infty} V_i(x) = V(x). \tag{16}$$