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PARTIAL INFORMATION IN NEWMAN'S REAL POKER

名古屋商科大　坂口　実(MINORU SAKAGUCHI)

ABSTRACT. Newman's 'real' poker is essentially different from the usual kinds of poker in that this allows arbitrarilly high bets. In [3] he proposed and actually proved, in an ingeneous way, a very interesting solution of this game and noted that the number 7 is mysteriously persent in this solution. In [4] the present author clarified the way through which this mystery appears. In the present paper we shall investigate Newman's real poker in which the first mover can obtain an additional information about whether his hand is lower or higher than his opponent's hand. It turns out, curiously enough, that the number 4 is present in the solution and the value of the game is 1/4 of the ante.

1. Two-Person Real Poker by Newman and Its Solution. Players I and II each ante 1 unit and are each dealt a "hand", namely, a randomly chosen real number in [0,1]. Each sees his, but not other's hand. First I bets any amount ($\geqq 0$) he chooses. Next II decides whether he sees the bet or folds. The payoff is as usual. Hence the rule of the game is described by the diagram (in which sgn $z \equiv 1$ $(z > 0)$, $0 (z = 0)$, or $-1 (z < 0)$):

Player	Hand	1st Move	2nd Move	Payoff to I
I	x	bet $\beta(x)$ →		
			fold	---------- 1
II	y		see	----- $(1+\beta(x))sgn(x-y)$

問題の定式化および解は略.

2. Partial Information in Newman's Real Poker (NRP). We introduce the notion of partial information into Newman's real poker. The Umpire tells one of the players about which is true $x < y$ or $x > y$. Let $J^{i:j}$ be the information structure such that

$$\left\{ \begin{matrix} i \\ j \end{matrix} \right\} = 1(0), \text{ if player } \left\{ \begin{matrix} \text{I} \\ \text{II} \end{matrix} \right\} \text{ dose (dosen't) get this information.}$$

This information is *partial* in the sense that one player is told which is true $x < y$ or $x > y$ only, and not told his opponent's hand. There are four information structres and the NRP explained in Section 1 is under the information structure $J^{0:0}$.

Let the value of the NRP under the information $J^{i:j}$ be denoted by $V(J^{i:j})$. Then the following result is straightforward.

Theorem 1. $V(J^{1:1}) = V(J^{0:1}) = 0.$

The purpose of this paper is to derive the solution to NRP under the information structure $J^{1:0}$ and to show $V(J^{1:0}) = 1/4$. Therefore together with the Newman's result mentioned in Section 1, we now have

$$V(J^{0:1}) = V(J^{1:1}) = 0 < V(J^{0:0}) = 1/7 < V(J^{1:0}) = 1/4. \tag{2.1}$$

The payoff function for our problem is

$$K(\beta_1, \beta_2 ; y_0) = 1 - \iint_{x<y \& y>y_0(\beta_1(x))} (2+\beta_1(x))dxdy + \iint_{x>y>y_0(\beta_2(x))} \beta_2(x)dxdy.$$

The routine of deriving the solution to this poker is:
1°) solve a finite discrete analogue of the NRP under the information $J^{1:0}$, 2°) on the basis of its solution, construct a conjecture on the form of the solution to our poker, and finally 3°) prove that the conjecture holds true.

3. A Finite Discrete Analogue of NRP under the Information $J^{1:0}$.

4. Passage to the Infinite Continuous Model

Theorem 3. *For the* NRP *under the information* $J^{1:0}$ *with payoff function* (2.2) *is as follows: An optimal strategy for* I, *when his hand is* x, *is to bet*

$$\beta_1^*(x) = \begin{cases} \text{a unique root } \beta \text{ in } (0,\infty) \text{ of the equation} & \\ 3\xi^2 - 2\xi^3 = 1 - 4x, \text{ where } \xi = 2/(2+\beta), & \text{if } 0 \leqq x < 1/4, \\ 0, & \text{if } 1/4 < x \leqq 1, \end{cases}$$

if $x < y$ *becomes known;*

$$\beta_2^*(x) = \begin{cases} 0, & \text{if } 0 \leqq x < 1/4 \\ \sqrt{3/(1-x)} - 2, & \text{if } 1/4 < x \leqq 1 \end{cases}$$

if $x > y$ *becomes known.*

The optimal strategy for II *is to see the I's bet* β, *if and only if his hand* y *exceeds* $y_0^*(\beta) = 1 - (3/2)/(2+\beta)$. *The value of the game is* $1/4$.

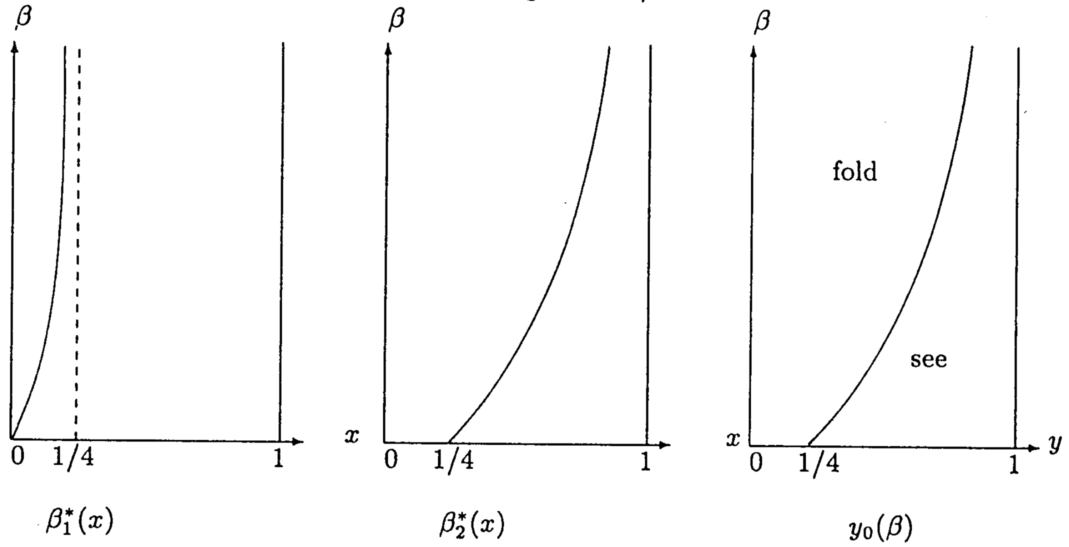

Fig. 2 An Optimal strategy-pair in NRP under $J^{1:0}$

5. Proof of Theorem 3

6. Final Remark

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3. D. J. Newman, A model for 'real' poker, Operations Reserch, 7 (1959). 557-560.

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7. M. Sakaguchi, Information structures and perfect information in some two-person poker, Math. Japonica. 38 (1993). 743-755.