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施設配置モデルにおける地図投影法の誤差

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1. Introduction

Although most demand data used in locational analysis lies on the surface of the earth, most of them are not spherical data but planar data obtained from a map produced by some type of map projection. As the earth is round and maps are flat, it is impossible to flatten the surface of the earth without stretching or tearing. In other words, the geometrical relationships on the surface of the earth cannot be perfectly duplicated, regardless of map projection. As a result, error is incurred if such planar location data are used. Accordingly, the conclusions drawn using planar location data are of questionable validity. In this paper, such an error is termed a *projection error*. Although examining projection error is of importance to decision-makers, little work has been done to theoretically clarify the characteristics of projection error.

The aim of this paper is to analyze how map projection affect the *location error*, defined as the difference between the optimal location using spherical data and that using planar data, as in the Weber model. Specifically, we examine the projection error of the maps projected by equirectangular projection when all the demand points lie on the Northern Hemisphere. This projection, which is classified as cylindrical projections, is very simple to construct and to interpret. In addition, this projection, sometimes called the cylindrical equidistant projection with two standard parallels, is composed of an evenly spaced network of horizontal parallels and vertical meridians, and has the added benefit that the scale is true on all meridians and on two parallels. This projection is good for city maps or estate maps.

2. Properties

Figure 2 presents the orthographic projection as centered on the spherical Weber point.

Lemma 1. $\{\phi^*, \lambda^*\}$ *is a local minimizer to the spherical Weber model if and only if* $(0,0)$ *is the optimal point to the planar model obtained by the orthographic projection centered on* $\{\phi^*, \lambda^*\}$. *This Lemma states that the minimizer to the planar Weber model on the map shown in Figure 2 becomes the center of the map.*

Moreover, on the orthographic projection map, any great circle arc through the center appears as a straight line segment; see Figure 1. In contrast to this, on the equirectangular projection map, any great circle arc through the center on the Northern Hemisphere becomes a concave curve, irrespective of selecting standard parallels; see Figures 2. This is because the equirectangular projection is classified as cylindrical projections.

Lemma 2. *On the equirectangular projection map, the locus of the greatest circle arc through the center on the Northern Hemisphere appears as a concave curve, irrespective of selecting standard parallels.*

We call the demand data set $\{\phi_i, \lambda_i\}, (i \in I)$ *symmetrical on* $\{\phi^*, \lambda^*\}$ if $|I|$ is even and for any $i \in \{1, \cdots, \frac{|I|}{2}\}$ $\omega_i = \omega_{\frac{|I|}{2}+i}$, and the shorter great circle arc connecting the i-th demand point and the $\frac{|I|}{2} + i$-th demand point also passes $\{\phi^*, \lambda^*\}$. As is evident from the definition, $\{\phi^*, \lambda^*\}$ becomes a local minimizer to the spherical Weber model. From Lemma 2, on the equirectangular projection map, the locus of greatest circle arc connecting the i-th and the $\frac{|I|}{2} + i$-th demand points, and passing through $\{\phi^*, \lambda^*\}$ become a concave curve. Because of this concavity, if the data set is symmetrical it follows that the sum of the sines on the orthographic projection map are greater than that of the equirectangular projection. Therefore, this together with Lemma 1 yields the following proposition:

Property 1. *For the data* $\{\phi_i, \lambda_i\}, (i \in I)$ *which are symmetrical on* $\{\phi^*, \lambda^*\}$, *then the southern* *direction at* (x^*, y^*) *becomes a decent direction in terms of the planar Weber model obtained by the equirectangular projection with parallels at* θ, *irrespective of* $(0 <)\theta(< \frac{\pi}{2})$.

Figure 3 shows the two types of locations for these capitals obtained by merging Figures 1 and 2, such that the direction and the projection of the spherical Weber point on Figure 1 coincide with those on Figure 2. In the latter Figure, the capitals on the orthographic projection, and the equirectangular projection are connected with the spherical Weber point by solid lines, and dotted lines, respectively. We see that in each city the solid lines always lie above the dotted lines. Accordingly, we would expect that if the standard parallels coincide with the latitude of the spherical Weber point, the sum of the sines on the orthographic projection map is greater than that of the equirectangular projection.

Lemma 3. *For the locus of the greatest circle arc connecting any two points on the Northern Hemisphere, its projection on the orthographic projection map lies above its projection on the equirectangular projection map with the standard paralleles at the latitude of one point, when the direction and the projection of the point on the former map coincide with the direction and the projection of the point on the latter map.*

Therefore, Lemma 3 together with Lemma 1 gives the following Property.

Property 2. *In terms of the planar model obtained by the equirectangular projection with the standard paralleles at* θ^*, *then the southern direction at* (x^*, y^*) *becomes decent direction in terms of the planar Weber model obtained by the equirectangular projection with parallels at* θ, *irrespective of* $(0 < \theta < \frac{\pi}{2})$.

References

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Figure 1: Map on an Equirectangular Projection

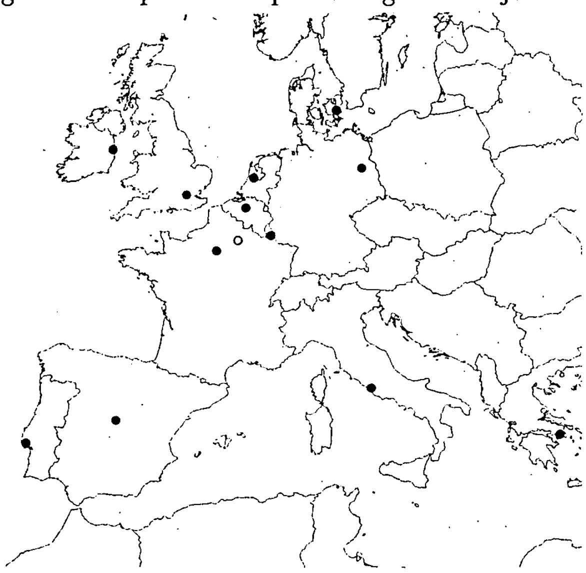

Figure 2: Map on an Orthographic Projection

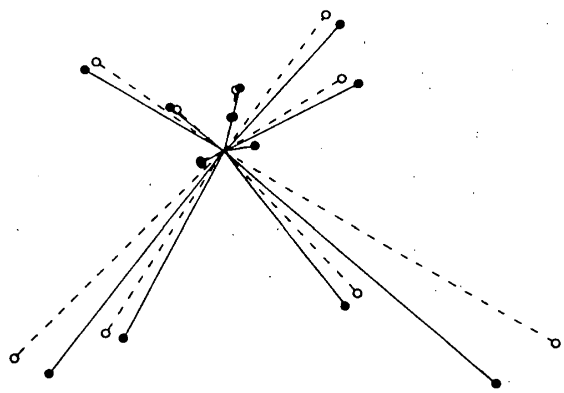

Figure 3: Two Types of the Locations of Capitals