1996年度日本オペレーションズ・リサーチ学会
春季研究発表会

地域間の消費税競争モデル

01009486　筑波大学社会工学系　大澤義明 OHSAWA Yoshiaki

1. Introduction

To our knowledge, most existing papers discussing spatial competition model the interaction between firms' decisions. However, governments compete with each other for revenues. Specific examples are the taxes on retail trade imposed by governments. In Europe, with the establishment of the European Community, the European market has become more open. This gives more healthy competition among the European countries as is the case among the states in America. The sales tax in Luxembourg is relatively low compared with Belgium, France and Germany. Accordingly, many people from Belgium, France and Germany go shopping to Luxembourg to seek a good bargain. In addition, most tourists passing through Luxembourg gas up their cars at gas stations there. This is because the gasoline taxes of Luxembourg are low, making the cost of gasoline in Luxembourg the cheapest in Europe. By underselling their competitors, both the government in Luxembourg are able to obtain a lot of revenues and make high profits. As, in the United States, many people go shopping to other states to obtain good bargains because the sales taxes are different between states . For many states sales taxes are among the principal sources of revenues. As a result, the state governments regard the strategy of price controls such as local taxes as important.

Obviously, a decrease in the tax of a government extends its market area. However, the revenues obtainable from its firms decreases. Thus governments face a trade-off, and attempt to make this trade-off effective. The spatial competition situation analyzed here differs from the standard version of the spatial competition model only in that the governments which divide the whole market segment compete by optimizing its government revenues.

The objective of this paper is to formulate the spatial competition model of imposing a tax between $N(\geq 2)$ governments on a line segment, and to examine the relationships between the revenues in equilibria and the spatial arrangements of the governments and between the revenues in equilibria and the sizes of the governments. More specifically, we shall compute all taxes in equilibria for some sets of geographical and technological parameters.

2. The model for ordinary firms

Consider the line segment along which customers and firms are evenly spread with a unit density. There are $N(\geq 2)$ governments which divide this whole market segment into N line segments. They are indexed by $1, \cdots, N$. There is a single homogeneous commodity that is produced by all firms at zero marginal production cost. The assumption of zero marginal production cost is introduced here so as to simplify the analysis; however, our results do not depend on it. Each government demands a tax, denoted by p_i, from the firms within it. Each customer buys the commodity from the firm offering the lowest *full price*, defined as the mill price plus the transport cost between the firm and the customer, irrespective of its full price. Thus, the demand is perfectly inelastic. Transport costs are linear in distance and equal to γ per unit distance. It should be noted that all firms in the i-th government would charge the same and constant mill prices, p_i. This is because as the firms compete with each other for customers, given the continuum of firms all firms would price at marginal cost.

We define the *revenue* of the i-th government as the sum of the taxes from all firms within it, referred to as $\pi_i(p_1, \ldots, p_N)$. And we define the *total demand* of the i-th government as the sum of the demands of all the firms within it, referred to as $D_i(p_1, \ldots, p_N)$. Then each government maximizes its revenue by changing its tax, assuming that it considers the others' taxes as fixed. In this paper, we define a tax equilibrium by a Nash equilibrium of a non-cooperative N-person game whose players are governments, strategies are taxes and payoffs are revenues. Let the tax and the size for the i-th government be denoted by p_i and $L_i(>0)$ respectively. As usual, a vector $(p^*_1,\ldots,p^*_N)$ is a *tax vector in equilibrium* if and only if $\pi_i(p^*_1,\ldots,p_i,\ldots,p^*_N) \leq \pi_i(p^*_1,\ldots,p^*_i,\ldots,p^*_N)$ for any $p_i \geq 0$ and $i=1,\cdots,N$. To make our notation simpler, we denote $\pi_i(p^*_1,\cdots,p^*_N)$ by $\pi_i{}^*$ and $D_i(p^*_1,\cdots,p^*_N)$ by D^*_i.

Proposition 1. *When the number of governments is two, there exists a unique tax vector in equilibrium as follows:*

$$p^*_1=\frac{\gamma}{3}(2L_1+L_2), \qquad p^*_2=\frac{\gamma}{3}(L_1+2L_2).$$

Proposition 2a. *When the number of governments is three and the sizes of the peripheral governments are the same, i.e., $L_1=L_3(\equiv L)$, there exists a unique tax vector in equilibrium given by*

$$\begin{aligned} p^*_1 &= \frac{\gamma}{6}(4L+L_2), \qquad p^*_2=\frac{\gamma}{6}(2L+2L_2),\\ p^*_3 &= \frac{\gamma}{6}(4L+L_2). \end{aligned}$$

Proposition 2b. *When the number of governments is three and the sizes of the adjoining governments are the same, i.e., $L_2=L_3(\equiv L)$, there exists a unique tax vector in equilibrium if and only if $L_1 \leq \frac{4\sqrt{13}+1}{3}L(\approx 5.141L_1)$. Whenever it exists, it is given by*

$$\begin{aligned} p^*_1 &= \frac{\gamma}{12}(7L_1+3L), \qquad p^*_2=\frac{\gamma}{12}(2L_1+6L),\\ p^*_3 &= \frac{\gamma}{12}(L_1+9L). \end{aligned}$$

Proposition 3. *When the sizes of all governments are the same, there exists a unique tax vector in equilibrium. It is given by*

$$p^*_i=\frac{\gamma L}{A(N)}\sum_{k=1}^{N}\min\{\alpha_k,\alpha_i\}\min\{\alpha_{N+1-k},\alpha_{N+1-i}\}.$$

where,
$\alpha_k \stackrel{\mathrm{def}}{=} \frac{1}{2}\{(2+\sqrt{3})^{k-1}+(2-\sqrt{3})^{k-1}\}, A(N) \stackrel{\mathrm{def}}{=} \alpha_N+2\sum_{k=1}^{N-2}\alpha_{N-k}+\alpha_1.$

Proposition 4. *For the transportation firms and $L_1 \geq L_2$, there exists a unique tax vector in equilibrium if and only if*

$$L_1 \geq L_2+3\sqrt{TL_1}-2T.$$

Whenever it exists, it is given by:

$$p_1{}^*=\frac{\gamma}{3}(2L_1+L_2-2T), \qquad p_2{}^*=\frac{\gamma}{3}(L_1+2L_2-T).$$

3. Conclusions

In summary, the following conclusions can be drawn from the preceding discussion:

(c1) When two government compete for revenues, for ordinary firms a Nash equilibrium necessarily and uniquely exists. For transportation firms the existence of a Nash equilibrium depends on the spatial configurations of governments.

(c2) The taxes and the revenues in equilibrium that we have obtained suggests that the spatial configuration and the sizes of contries are essential for governments. This is because, for interior governments, encroachments on the market areas of neighbours become profitable to the government, thus making the competitive process more stringent. While, the peripheral government enjoys a local monopoly.

(c3) Considering the size ratios of countries, smaller countries, like Luxembourg, will generate more revenues than bigger countries like France and Germany when the European market is opened. This conclusion is more apparent when the length of trips is increased.

References

[1]Andeson, S.P., A. de Palma, and J.-F. Thisse, 1992, *Discrete Choice Theory of Product Differentiaion*(MIT Press, Cambridge).

[2]d'Aspremont, C., J. J. Gabszewicz and J.-F. Thisse, 1979, On Hotelling's Stability in Competition, *Econometrica, 47*, 1045-1050.

[3]Gabszewicz, J.J and J.-F. Thissse, 1992, Location. In R.J.Aumann, S.Hart(eds.) *Handbook of Game Theory with Economic Applications* (North-Holland, Amsterdam).