1996年度日本オペレーションズ・リサーチ学会
春季研究発表会

Airline Seat Management with Multiple Flight-legs

会員番号：02991570　　所属：筑波大学　　氏名：遊 鵬勝　You Peng-Sheng

Abstract It is common for airline companies to classify a pool of identical seats on the same flight into several booking classes and charge different fares. This paper will try to develop a dynamic programming model which brings about selection rule and pricing policy for flights of multiple flight-legs with multiple booking classes.

1 Instruction

Airline market is a highly competitive market. In such a competitive market, competition has been focused on quality of service, pricing policy, route structure and so on. In order to attract customers with different purchasing power, it is common for airline companies to classify a pool of identical seats on the same flight into several booking classes through the application of restrictions or service on tickets.

A number of papers [1,2,4,6] have focused on the problem of single flight-leg with multiple fare classes. WONG [5] have developed a model for multiple flight-legs with single fare class by using the flexible assignment approach, which led to develop rules for trip seat allocation. DROR [3] has presented a mathematical approach (maximal flow) allocating seats to different categories of customers.

This paper develops a discrete-time dynamic programming model which brings about a decision rules (selection of customers and pricing policy) for the problem associated with multiple flight-legs with multiple booking classes. From this model, we will try to develop a selection rule which determines whether to accept or deny an arriving customer who requests for a certain booking class of a trip and a pricing policy which determines the appropriate price to be offered to a customer once the airline company decides to accept the customer.

- *A different Concept*

The problem of seat booking management is highly affected by customers' arriving patterns and permissible purchasing powers. The assumptions of previous researches are that all accepted customers must purchase tickets. However, in most cases, customers will request for price quotations before they buy their tickets and may switch to other companies if they decide that the prices offered by the airline are too high. Therefore, it is not likely that all accepted customers will purchase their tickets.

2 Model

Consider the following discrete time sequential stochastic decision process with a finite planning horizon. First, for convenience, let points in time be numbered backward from the final point in time of the planning horizon as t, $t-1$,... and so on, where the interval between two successive points in time, say time t and time $t-1$, is period t. Here, the period is small enough that no more than one request (from any booking class in any trip) arrives and no more than one flight departs during that period. In this paper, let us denote the following notations:

Notations

j	airport. $0 \le j \le N$
$d_{j,j+1}$	a flight-leg from airport j to $j+1$.
$i_{j,j+1}$	seats available for flight-leg $d_{j,j+1}$.
f_{jk}	a trip from airport j to k.
s_{jk}	seats available for trip f_{jk}. $s_{jk} = \min\{i_{m,m+1} \mid m = j, \cdots, k-1\}$
i	a set with elements that are the number of seats available for all flight-legs, that is $i = \{i_{01}, i_{12}, \cdots, i_{N-1,N}\}$. Let $0 = i$ with $i_{j,j+1} = 0, j = 0, 1, \cdots, N-1$.
$S(i)$	the set of all trips in which the remaining seats available is no less than 1. $S(i) = \{f_{jk} \mid s_{jk} \ge 1, 0 \le j < k \le N\}$.
L_{jk}	the number of booking classes of a trip f_{jk}.
t_j	departure time from airport j where $0 = t_{N-1} < t_{N-2} < \ldots < t_1 < t_0$.
h_t	the present or the next airport to depart from at time t, that is, $h_t = \min\{j \mid t_j \le t, j = 1, \cdots, N-1\}$
c_{jkl}	cost for carrying a passenger of class l of f_{jk}.
β	per-period discount factor, $\beta \in (0, 1]$.

$\lambda_t^l(f_{jk})$ the probability that a customer of class l in trip f_{jk} arrives at time t. Let $\lambda_t^0 = 1 - \sum_{j=h_t}^{N-1} \sum_{k=j+1}^{N} \sum_{l=1}^{L_{jk}} \lambda_t^l(f_{jk})$ be the probability that no customer arrives.

Furthermore, let $p_{jkl}(x)$ be the probability that a customer of class l of f_{jk} buys a ticket if the offered selling price is x. Here, for given a_{jkl} and b_{jkl} such that $0 \le a_{jkl} < b_{jkl}$, let $p_{jkl}(x) = 1$ for $x \le a_{jkl}$, $p_{jkl}(x) = 0$ for $b_{jkl} \le x$, and $p_{jkl}(x)$ be strictly decreasing in x for $a_{jkl} \le x \le b_{jkl}$. Let $P_{jkl}(\theta)$ be the probability distribution function of the maximum permissible purchasing price θ that a customer of class l of f_{jk} has in mind for the ticket. That is, if the offered price is smaller than θ, then the customer will decide to buy. In this case $p_{jkl}(x)$ can be given by

$$p_{jkl}(x) = \int_x^\infty dP_{jkl}(\theta). \tag{2.1}$$

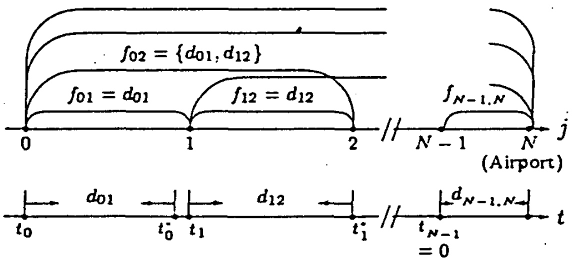

Figure 1

3 Formulation

Let $v_t(i)$ denote the maximum total expected present discounted profit starting from time t with i seats available for reservation. Then, clearly, we have

$$v_t(0) = 0, \qquad t \ge 0, \tag{3.1}$$

$$\begin{aligned} v_t(i) &= \lambda_t^0 \beta v_{t-1}(i) \\ &+ \sum_{\substack{j=h_t \\ f_{jk} \notin S(i)}}^{N-1} \sum_{k=j+1}^{N} \sum_{l=1}^{L_{jk}} \lambda_t^l(f_{jk}) \beta v_{t-1}(i) \\ &+ \sum_{\substack{j=h_t \\ f_{jk} \in S(i)}}^{N-1} \sum_{k=j+1}^{N} \sum_{l=1}^{L_{jk}} \lambda_t^l(f_{jk}) \max\{g_t^l(i, f_{jk}), \beta v_{t-1}(i)\} \quad t \ge 1, \end{aligned} \tag{3.2}$$

where $g_t^l(i, f_{jk})$ is the maximum total expected present discounted profit starting from time $t \ge t_j$ with i seats remaining, provided that the airline accepts the customer of class l of trip f_{jk} who has just arrived and decides to offer a price to the customer. Then $g_t^l(i, f_{jk})$ can be expressed as

$$\begin{aligned} g_t^l(i, f_{jk}) = \max_x \{ & p_{jkl}(x)(x - c_{jkl} + \beta v_{t-1}(i_{jk})) \\ & + (1 - p_{jkl}(x)) \beta v_{t-1}(i)\}, \end{aligned} \tag{3.3}$$

where

$$e_{jk} = \{\underset{1}{0}, \underset{2}{0}, \cdots, \underset{j+1}{1}, \underset{j+2}{1}, \cdots, \underset{k}{1}, \underset{k+1}{0}, \underset{k+2}{0}, \cdots, \underset{N}{0}\}, \tag{3.4}$$

$$\begin{aligned} i_{jk} = i - e_{jk} = \{ & i_{01}, i_{12}, \cdots, i_{j,j+1} - 1, i_{j+1,j+2} - 1, \\ & \cdots, i_{k-1,k} - 1, i_{k,k+1}, \cdots, i_{N-1,N}\}. \end{aligned} \tag{3.5}$$

The final condition is given as follows:

$$v_0(i) = \sum_{\substack{l=1 \\ f_{N-1,N} \in S(i)}}^{L_{N-1,N}} \lambda_0^l(f_{N-1,N}) \max\{g_0^l(i, f_{N-1,N}), 0\} \tag{3.6}$$

where

$$g_0^l(i, f_{N-1,N}) = \max_x p_{N-1,Nl}(x)(x - c_{N-1,Nl}). \tag{3.7}$$

4 Conclusion

In this paper, by $x_t(i, f_{jk}, l)$ we denote the smallest optimal price if more then one exist. The conclusions obtained in this paper are summarized as follows:

1. When the number of flight-legs is less than or equal two ($N \le 2$), thenthe optimal price $x_t(i, f_{jk}, l)$ is monotone function of remaining seats and remaining decision periods if $\beta = 1$.
2. When the number of flight-legs is equal to or more than three ($N \ge 3$), the optimal price $x_t(i, f_{jk}, l)$ is not always monotone in t or i.

REFERENCES

[1] P. P. BELOBABA, "Application of a Probabilistic Decision Model to Airline Seat Inventory Control," *Operations Research* 37, 183-197 (1989).

[2] S. L. BRUMELLE AND and J. I. MCGILL "Airline Seat Allocation with Multiple Nested Fare Classes," *Operations Research* 41, 127-137 (1993).

[3] M. DROR, P. TRUDEAUN AND S. P. LADANY, "Network Models for Seat Allocation on Flights," *Transportation Research* B 22B, 239-250 (1988).

[4] T. C. LEE AND M. HERSH, "A Model for Dynamic Airline Seat Inventory Control with Multiple Seat Bookings," *Transportation Science* 27, 3, 252-265 (1993).

[5] J. T. WONG, F. S. KOPPELMAN AND M. S. DASKIN, "Flexible Assignment Approach to Itinerary Seat Allocation," *Transportation Research B* 27B, 33-48 (1993).

[6] R. D. WOLLMER, "An Airline Seat Management Model for a Single Leg Route when Lower Fare Classes Book First," *Operations Research* 40, 26-37 (1992).