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An Internationally Diversified Investment Using an Integrated Portfolio Model

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1. Introduction

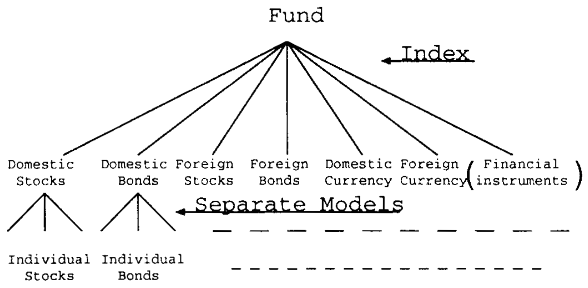
The correlation among securities of foreign countries is sometimes significantly less than the correlation among securities of the same country. This suggests that portfolios that consist of securities of several countries should have less risk than those consisting of securities of a single country.

In this paper we use a new integrated portfolio model which takes care of stocks and bonds of several countries to construct an internationally diversified portfolio. This serves as an alternative to the popular asset allocation strategy, in which the fund is first allocated to indexes corresponding to diverse asset classes and then allocated to individual assets using appropriate models for each asset class. Our model, on the other hand, determines the allocation of the fund to individual assets in one stage by solving a large scale mean-variance or mean-absolute deviation model.

Computational experiments show that the new approach can serve as a more reliable and less expensive method to allocate the fund to diverse classes of assets.

2. Asset Allocation Strategy

Suppose that the fund is going to be allocated to diverse classes of assets including stocks, bonds, derivatives (both domestic and foreign), currencies, real estates etc. . Let there be m asset classes and let the j-th asset class $C_j (j = 1, \cdots, m)$ consist of n_j individual assets. The eventual goal is to determine the amount of fund x_{jk} to be allocated to k-th asset in C_j to minimize the risk while maintaining a specified expected rate of return.



Asset Allocation

In standard asset allocation strategy, one first determines the proportion of fund x_j^* to be allocated to asset class C_j by solving index MV model and then determines x_{jk} by solving individual MV model:

$$
\begin{array}{|ll}
\text{minimize} & V\left[\sum_{k=1}^{n_j} R_{jk}x_{jk}\right] \\
\text{subject to} & E\left[\sum_{k=1}^{n_j} R_{jk}x_{jk}\right] = r_j \\
& \sum_{k=1}^{n_j} x_{jk} = x_j^*, \\
& x_{jk} \geq 0, k = 1, \ldots, n_j
\end{array}
$$

3. Mean-Variance and Mean-Absolute Deviation Models for International Diversified Stock-Bond Portfolio Optimization Model

Let us consider the allocation of fund to several classes of assets of m different countries. Let E_i and F_i be, respectively, the currency exchange rate and the forward exchange rate between the domestic currency and the currency of country i. Also, let S_{ij} $(i = 1, \ldots, n_i)$ be the amount of fund to be allocated to the asset j of country i and let Y_i be the amount of forward contract on currency of country i. Further, let $\tilde{E}_i$, $\tilde{S}_{ij}$, $\tilde{Y}_i$ be, respectively, the future value of E_i, S_{ij} and Y_i at the end of the period.

Then total wealth W at the first of the period is given by $W = \sum_{i=1}^{m}\sum_{j=1}^{n_i} E_i S_{ij}$ respectively, the total wealth $\tilde{W}$ at the end of the period is given by $\tilde{W} = \sum_{i=1}^{m}(\tilde{E}_i \sum_{j=1}^{n_i} \tilde{S}_{ij} + F_i Y_i - \tilde{E}_i Y_i)$

Let

$$y_i = Y_i / \sum_{j=1}^{n_i} S_{ij}, \quad w_i = \sum_{j=1}^{n_i} E_i S_{ij} / \sum_{i=1}^{m}\sum_{j=1}^{n_i} E_i S_{ij},$$
$$f_i = F_i/E_i - 1, \quad x_{ij} = S_{ij} / \sum_{j=1}^{n_i} S_{ij},$$
$$\tilde{e}_i = \tilde{E}_i / E_i - 1, \quad \tilde{r}_{ij} = \tilde{S}_{ij}/S_{ij} - 1.$$

We follow the standard mean-variance (MV), or the mean-absolute deviation (MAD) framework and define the problem as follow:

$$\begin{array}{|l}
\text{minimize} \\
f(\sum_{i=1}^{m}\{\sum_{j=1}^{n_i}(\tilde{e}_i + \tilde{r}_{ij} + \tilde{e}_i\tilde{r}_{ij})v_{ij} - (\tilde{e}_i - f_i)z_i\}) \\
\text{subject to} \\
\sum_{i=1}^{m}\{\sum_{j=1}^{n_i}(e_i + r_{ij} + \sigma_{ij} + e_i r_{ij})v_{ij} - (e_i - f_i)z_i\} \\
\quad \geq \rho \\
\sum_{i=1}^{m} w_i = 1, \ \sum_{j=1}^{n_i} v_{ij} = w_i, \quad i = 1, \ldots, m \\
l_i w_i \leq z_i \leq u_i w_i, \quad i = 1, \ldots, m \\
w_i \geq 0, \ v_{ij} \geq 0, \quad j = 1, \ldots, n_i; \ i = 1, \ldots, m
\end{array}$$

4. Comparison of Asset Allocation and Integrated Approach

Suppose that the fund is invested in Japan stocks (212 of NIKKEI 225), Japan bonds (31), U.S. stocks (95 of S&P 100) and U.S. bonds (7 indices). The total number of assets is $n_1 + n_2 = 345$. Also, we employ the MAD model using historical monthly stock/bond data during 84 months from January, 1989 to December,1995.

Figure 1 and 2 show the typical result of simulation.

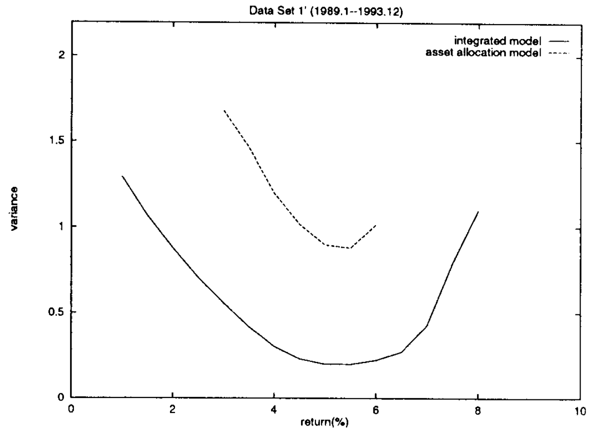


Figure 1. comparison of efficient frontier

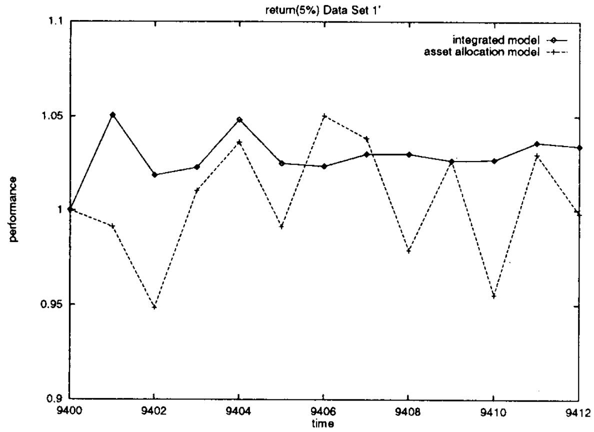


Figure 2. comparison of performance

We see from these figures that the integrated model is better than asset allocation model as expected.

5. Reference

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