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Two-level Optimal Design Problems for Distribution Systems

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1 Introduction

We consider two optimal design problems of distibution systems. Each problem has a two-level structure such that at the lower level goods are supplied to customers with known demands in such way as to minimize the routing cost. On the other hand, the upper level determines the configuration of the distribution system, such as the location of depots, the number of vehicles assigned to each depot, or the assignment of customers to appropriate depots, so as to minimize the total cost of the whole system. We formulate the following two types of the problem.

Problem I: Among potential locations of depots each of which possesses its own fixed number of vehicles, the upper level chooses N_D locations to be used as depot sites. The lower level then finds optimal routes with minimum cost from these selected depots to customers. The problem thus becomes a two-level combinatorial optimization problem whose upper and lower level problems are an allocation problem and a *vehicle routing problem* (VRP) with multi depots, respectively.

Problem II: The upper level can entrust to N_D distributers to distribute goods. The task of the upper level is to partition the set of customers into N_D groups and assign each group to an appropriate distributer. At the lower level, each distributer solves standard (single depot) VRP to find a minimal cost route travelling the assigned customers. Consequently, in this problem, the upper level problem is a set partitioning problem, and the lower one is a set of VRP's.

Both problems can be formulated in a framework of general two-level optimization problem. Let $\boldsymbol{x}, \boldsymbol{y}$ be decision variables of the upper level problem (ULP) and the lower level problem (LLP) respectively. Then we define the ULP and the LLP of this distribution problem as follows :

$$\begin{array}{llll} \text{(ULP)} & \min\limits_{\boldsymbol{x}} & F(\boldsymbol{x}, R(\boldsymbol{x}^*)) & \\ & \text{s.t.} & \boldsymbol{x} \in X, & \\ \text{(LLP)} & & R^* = \min\limits_{\boldsymbol{y}} & f(\boldsymbol{x}, \boldsymbol{y}) \\ & & \quad\quad \text{s.t.} & \boldsymbol{y} \in Y(\boldsymbol{x}) \end{array}$$

where X and $Y(\boldsymbol{x})$ denotes the feasible set of the ULP and the set of feasible routes under given $\boldsymbol{x}$, respectively, and $R^*(\boldsymbol{x})$ stands for the minimum total cost of the LLP.

2 Problem formulation

Formulation of LLP[1]

Under given $\boldsymbol{x}$ (the configuration parameters), the LLP is a VRP with multi depots (Problem I) or a set of VRP's with single depot (Problem II). We slightly extend the VRP formulation by Fisher [1] to the case of multi depots as follows.

A graph $G = (N, E)$ is defined by the set N of its nodes and the set E of its edges. We also define $C = \{1, \ldots, N_C\}$: set of customers, N_C : number of customers; $D = \{N_C + 1, \ldots, N_C + N_D\}$: set of depots, N_D : number of depots; $N = \{1, \ldots, N_C, N_C + 1, \ldots, N_C + N_D\}$: set of nodes; k_d : number of vehicles at depot d; $V = \{1, \ldots, k_1, k_1 + 1, \ldots, k_1 + k_2, \ldots, K\}$: set of vehicles, $K = \sum_{d=1}^{N_D} k_d$: number of vehicles; Q : capacity of each vehicle; q_i : quantity of goods to be delivered to customer i; c_{ij} : cost of direct travel between node i and j $i \in C, j \in N$, $c_{ij} = c_{ji}$ for all $i, j \in C$.

For $S \subseteq C$, let $\bar{S} = N \setminus S, d(S) = \sum_{i \in S} q_i$ and $r(S) = \lceil d(S)/Q \rceil$, where $\lceil a \rceil$ denotes the smallest integer not less than a. It is noted that

$r(S)$ stands for the minimal number of vehicles that required to serve customers in set S, so as to meet their demands.

Let $y_{ijk} = 1$ if vehicle $k \in V$ uses the edge (i,j) $i \in C, j \in N, y_{ijk} = 0$ otherwise. Since the edge (i,j) is undirected, y_{ijk} and y_{jik} denote the same variable. We also define
$Y = \{y_{ijk} \mid y_{ijk} = 0$ or 1 and defines a K-tree satisfying $\sum_{k=k_{d-1}+1}^{k_d} \sum_{i=1}^{N_C} y_{i(N_C+d)k} = 2k_d,\ d \in D, (k_0 = 0)\}$,
where K-tree is defined to be a graph with $N_C + N_D + K$ edges spanning all nodes. Then the problem is

$$\min_{\boldsymbol{y} \in Y} \quad R = \sum_{i=1}^{N_C} \sum_{j=i+1}^{N_C+N_D} \sum_{k=1}^{K} c_{ij} y_{ijk} \tag{1}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} \sum_{j=1}^{N_C} y_{ijk} = 2, \quad i \in C \tag{2}$$

$$\sum_{i=1}^{N_C} y_{ipk} - \sum_{j=1}^{N_C} y_{pjk} = 0, \quad k \in V, p \in C \tag{3}$$

$$\sum_{k=1}^{K} \sum_{i \in S} \sum_{j \in \bar{S}} y_{ijk} \geq 2r(S) \quad \forall\ S \subseteq N_C \text{ with } |S| \geq 2 \tag{4}$$

Constraint (2) imposes that each customer has two edges incident on itself. It means that each customer must be visited exactly once. This constraint, however, does not necessarily imply that visiting vehicle and leaving vehicle are identical. Thus we add constraints (3) for each vehicle. Inequalities (4) denote the vehicle capacity constraints. Since each vehicle must enter and leave set S, we must have at least $2r(S)$ edges between sets S and $\bar{S}$.

If we set $N_D = 1$, the above problem reduces to a standard VRP studied in [1].

Formulation of ULP I

We assume that the number of depots and vehicles to be set are given. We define additionally $P = \{1, \ldots, N_P\}$: set of potential locations, N_P : number of potential locations; p_{ij} : fixed cost of placement of depot i to location j.

Let $x_{ij} = 1$ if depot i is located at location j, $x_{ij} = 0$ otherwise. Then the problem is

$$\min_{\boldsymbol{x}} \quad Z = \sum_{i=1}^{N_D} \sum_{j=1}^{N_P} p_{ij} x_{ij} + R^*(\boldsymbol{x}) \tag{5}$$

$$\text{s.t.} \quad \sum_{j=1}^{N_P} x_{ij} = 1, \quad i \in D \tag{6}$$

$$\sum_{i=1}^{N_D} x_{ij} \leq 1, \quad j \in P \tag{7}$$

$$x_{ij} \in \{0,1\}, \quad i \in D,\ j \in P \tag{8}$$

Formulation of ULP II

Let $x_{ij} = 1$ if customer i is assigned to depot j and this means that customer i is served by the distributer at depot j, $x_{ij} = 0$ otherwise. Then the problem is

$$\min_{\boldsymbol{x}} \quad Z = \sum_{i=1}^{N_C} \sum_{j=1}^{N_D} p_{ij} x_{ij} + R^*(\boldsymbol{x}) \tag{9}$$

$$\text{s.t.} \quad \sum_{j=1}^{N_D} x_{ij} = 1, \quad i \in C \tag{10}$$

$$x_{ij} \in \{0,1\}, \quad i \in C, j \in D \tag{11}$$

3 Computational Results

Branch and bound type solution method and some computational results will be reported.

References

[1] M. L. Fisher, "Optimal Solution of Vehicle Routing Problems Using Minimum K-Trees," Operations Research, Vol. 42, 626-642, 1994.

[2] M. L. Fisher, "A Polynomial Algorithm for the Degree-Constrained Minimum K-Tree Problem," Operations Research, Vol. 42, 775-779, 1994.

[3] K. Shimizu, Y. Ishizuka, J. F. Bard, "Nondifferentiable and Two-level Programming," Kluwer Academic Publishers, 1997.