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Measuring Progress of an industry Using DEA

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1. Introduction

It is an important for company manager to know his/her campany's position in the industry. To know the information such as this, the manager compares the performance of his/her company with the standard of industry. He/she knows easily his/her present (or past) financial condition and so on, but difficultly the magnification of progress (or regress) of the entire industry which is belonged to the company. So, it becomes an effective information for the company to calculate and grasp the progress of the industry over periods.

Often, the ratio of the one year's gross sales to the previous year sales employs to represent the measurement of progress of the industry. But, this ratio may be represent incorrectly the degree of progress according to include some inefficient companies, because these company may be inefficient due to make their less effort.

Wei et al. [4] proposed a method based on DEA that is measured the technical progress over time. Their method can be solve only the *progress* case but not the *regress*. Further there are some problems in their method. In this study, we propose a index as measurement of the progress (or regress) of an entire industry based on and improved Wei et al.s' method.

2. Measuring progress or regress

Wei et al. proposed a method based on BCC model that could be measured the technical progress over periods, from 0 to t. They defined the ratio per unit period of the efficient frontier's shift over periods as the measurement of progress. When we can divide all the shift behaviors of the efficient frontier over periods into three cases, i.e. *progress*, *regress* and *intersect*. Only the *progress* cases can be obtained correctly the ratio by their method, because the DMU at period 0 is mapped on the efficient frontier at period t. Another problem in Wei et al.s' method is that if all the DMUs at period 0 is not included in the production possibility region at period t, then it may has some slack or not have any solution.

Generally, it is said that the production function is S in shape. Cobb-Douglas type production function which is one of S shape functions can be a linear function through logarithmic transformation. Also, if the optimal solution has some positive slack then incorrectly efficiency score. The constrained facet analysis (CFA) concept which extends the efficient frontier to outside the production possibility region is well-known method can be excluded the influence of the slack and solve above Wei et al.s' another problem. So in this study, we employ the multiplicative BCC model with CFA by Hirase et al. [3] to represent the production possibility region.

To solve the problem that if there are some slack, then the efficiency ratio is bend, we employ the Andersen and Petersens' model [1]. Most characteristic of their model is that we can obtain the magnification (or contraction) ratio to construct the efficient frontier without the object DMU.

2.1. Model

We formulate our model which has the production possibility region based on the multiplicative BCC model. It is assumed that we observed DMUs over period 0 through period t. We focus on the progress (or regress) ratio per unit period. So, we first calculate the progress ratio (or regress) overall periods. The notation '$\hat{}$' means logarithmically transformed, i.e. $\hat{X}_{ij}^t$ equals $\{\ln X_{ij}^t\}$.

[P1]

$$\begin{aligned} \max \quad & \hat{\gamma}_o \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_{jo} \hat{X}_{ij}^t \le \hat{X}_{io}^0, \quad i = 1, ..., m \\ & \sum_{j=1}^{n} \lambda_{jo} \hat{Y}_{ij}^t \ge \hat{\gamma}_o + \hat{Y}_{io}^0, \quad r = 1, ..., k \\ & \sum_{j=1}^{n} \lambda_{jo} = 1 \end{aligned}$$

$$\lambda_{jo} \geq 0, \quad j = 1, ..., n$$

where X_{ij}^q and Y_{rj}^q is the ith input and the rth output of DMU_j at period q, respectively and λ_{jo} is the nonnegative connection variable for DMU_j. In addition, we define that E^q is the set of efficient DMU at period q. The DMU in E^q is identified by solved the ordinal multiplicative BCC model for DMUs at period q, e.g. [2]..

If the optimal solution $\hat{\gamma}_o^*$ in [P1] is zero, it has a potential risk that DMU_o at period 0 is an element constructing the efficient frontier in [P1]. But the efficient frontier in [P1] must be constructed by only DMUs at period t. To overcome this risk, we apply Andersen and Petersen model to investigate DMU_o at period 0. If its solution is not 0, then DMU_o at period 0 is above the efficient frontier at period t. So, we can conclude that DMU_o is *regress* over periods.

Moreover, if there has a positive slack, we apply Hirase et al.s' model to exclude the influence of slack value.

2.2. Calculating progress ratio

γ_o^* is exponetially transformed $\hat{\gamma}_o^*$, i.e. γ_o^* equals $\exp\{\hat{\gamma}_o^*\}$.

Assume that Z_o^q is the performance of DMU_o at period q, then γ_o^* satisfies the following relation:

$$\frac{Z_o^t}{Z_o^0} = \gamma_o^* = (1 + \alpha_o)^t$$

Thus,

$$\alpha_o = \sqrt[t]{\gamma_o^*} - 1$$

We define the index α_o as the progress (or regress) ratio for DMU_o per unit period.

Also, we want to calculate the progress of the entire industry under the condition that the management of DMU performs efficient. The progress (or regress) ratio of the entire industry is defined from the set of the efficient DMUs at period 0, i.e. E^0. In short, the index for the entire industry means the ratio of the efficient frontier's shift. So, we define the index of entire industry as follow:

$$\alpha = \frac{1}{n^0} \sum_{o \in E^0} \alpha_o$$

where n^0 is the number of elements in E^0.

These indices is same as sense of Wei et al. If α_o or α is more (less) than unity, then it means the *progress* (*regress*), respectively.

2.3. Procedure

Step 1: Evaluate the efficient for each DMU for each period using the multiplicative BCC model [2], and identify the efficient DMU for each period.

Step 2: Solve [P2] for each DMU in the E^0 to DMUs at period t. If it is infeasible or there has a positive slack, then go to step Step 3. And if the optimal solution $\hat{\gamma}_o$ equals 0, then apply Andersen and Petersen model.

Step 3: Constructing the efficient frontier with E^t using Hirase et al.s' model. Then, evaluate the DMU in E^0 compared with this frontier.

Step 4: Calculate the indices, α_o and α, using the optimal values which are obtained by Step 2 and 3.

A numerical example will be reported in the presentation.

3. Conclusion

This paper discussed that the method to measure the progress (or regress) of an industry based on DEA methodology. Also we proposed two progress indices i.e. the progress (or regress) ratio for each company (DMU) and the industry (the set of DMUs). It may be a future work to compare our indices with some official (or public) indices about these progresses.

References

[1] Andersen, P. and N. C. Petersen: "A Procedure for Ranking Efficient units in Data Envelopment Analysis," *Management Science*, Vol39, pp.1261–1264 (1993).

[2] Charnes, A., W. W. Coorer, L. Seiford and J. Stutz: "A Multiplicative Model for Efficiency Analysis," *Socio-Economic Planning Sciences*, Vol.16, No.5, pp.223–224 (1982).

[3] Hirase, K., T. Yamaguchi, M. Goda and T. Fukukawa: "The Multiplicative DEA Model using CFA," Communications of the Operations Research Society of Japan, Vol.39, No.11, pp.613–618 (1994). in Japanese

[4] Wei, Q. L., B. Sun and Z. J. Xiao: "Measuring Technical Progress with Data Envelopment Analysis," *European Journal of Operational Reserch*, Vol.80, pp.691–702 (1995).