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Solving Linear Integer Optimization Problems by Linear Programming Cooperated with Constraint Programming

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1. Introduction

In all areas of industry and business, the optimization of resource allocation has become critical. Services and products must answer the increasingly specific needs of customers. This has led to an explosion in the number of products and services companies are obliged to provide their customers with. This, of course, also drives up costs for businesses and greatly increases the complexity of both problems and solutions. Optimization is, therefore, needed to control this ever-increasing complexity.

There are many different approaches to optimization. The best approach depends on the problem. Linear programming and its extension to integer programming are one of the most well-known optimization approaches. Constraint programming is a relatively new technique that has been proved particularly powerful when used to solve problems with complicated constraints. Both of them are widely used throughout industry and operations research and math programming communities.

The approach suggested by this paper is to combine constraint programming with linear programming to solve complex linear integer optimization problem more quickly.

2. Constraint programming

Constraint programming (CP) is a way of solving constraint satisfaction problems, which consists of a number of variables and a number of relations on and among those variables. A constraint is a mathematical relation between possible values of variables. The set of possible assignments of values to variables is known as the search space. Finding solution to a constraint-programming problem is to assign values to the constraint variables of the problem in such a way that all the constraints imposed on the variables are satisfied simultaneously. Constraint programming is efficient because, rather than searching that search space blindly, it exploits the constraints themselves by constraint propagation to reduce its effort in the search. The constraint propagation is known as the most powerful features of the constraint programming. However, the weakness of this approach becomes apparent when searching the feasible region for an optimal solution. If the feasible region is large, this approach may become inefficient at finding the best solution.

3. Integer linear programming

Linear programming (LP) is probably the best known and most widely used optimization technique. Any problem that can be formulated with real decision variables, a linear objective function, and linear constraint functions, may be solved using linear programming. This approach has been tremendously successful in solving a broad range of commercial resource allocation problems.

Integer linear programming uses linear programming to solve problems with some integer variables (all other variable types may be represented by a combination of integer variables and linear constraints), but still having linear objective and constraint functions. The integer variables are represented as real variables, and the resulting linear program is solved; then a repetitive process is used in which an integer variable is bounded above or below in an attempt to force it to an integer value by adding constraints, and the modified linear program is resolved. This method (branch and bound) terminates when all the integer variables take integer values. When the number of integer variables is small, integer programming solves problems fairly quickly; unfortunately, this process can be too time consuming for problems with large numbers of integer variables. Some problems require millions of iterations to be solved.

4. Cooperating linear programming with constraint programming

One approach to solve linear integer optimization problem is to combine linear programming with constraint programming. This approach may take advantages from both of them and speeds up the search for the solutions.

The way to cooperate them together is based on the traditional branch and bound procedures used to solve integer problems in linear programming. Basically, a linear relaxed problem is used to provide an approximation of the optimal solution. For a minimization (maximization) problem, this approximation provides a lower (upper) bound of the cost function. Then, the optimal relaxed solution computed by the linear programming can be used to guide the search procedure. First, the linear constraints in the problem to be solved are posted to both linear programming and constraint programming. Then, linear programming computes the optimal solution (if any) of the problem, and sends the optimal cost value to constraint programming, which updates the domain of the cost variable accordingly. Finally, a search procedure built with the search facilities of constraint programming may use the optimal relaxed solution provided by the linear programming as an approximation of the optimal solution. Late during the search, when the search procedure deduces or tries new bounds on a variable, constraint programming sends the new bounds to the linear programming, which updates the relaxed solution according to this bound modification and computes the new minimum (maximum) value of this cost function.

5. Case study

A multi-knapsack problem is used here as a case study to show the effect of cooperating linear programming with constraint programming. A knapsack problem can be defined like this, suppose we have a knapsack in which we want to put objects of different types. Each type of objects has a weight and a value. Since we can carry only a given weight, the knapsack can be packed only up to that limit. The objective is to put a set of objects in the knapsack so that the value of its contents is maximized. A multi-knapsack problem is an integer problem (that is, a problem with linear constraints on integer variables) with more than one constraint on the objects (e.g. not only type but also volume constraint). There are many industrial applications of such problems, such as capital budgeting, stock-cutting problems, resource planning and so forth.

For this problem, the weight constraint can be written as a linear inequality, and the objective function as a linear function, where all the constrained variables are integers. Thus, the problem can be described as,

*Maximize sum (j) c[j] * x[j]*
Subject to
  (1)Forall (i)
    *sum (j) a[i,j] * x[j] <= b[i];*
  (2)x[j] positive integers;
  (3)c[j] and a[i,j] positive
  (4) i=1,2,...,m;   j=1,2,...,n;

One sample data set is as follows,

```
n = 7;   m = 12;
b[j] = [ 18209, 7692, 1333, 924, 26638, 61188, 13360 ];
c[i] = [ 96, 76, 56, 11, 86, 10, 66, 86, 83, 12, 9, 81 ];
a[i,j] = [
 [19,  1, 10,  1,   1, 14,152, 11,  1,   1,  1, 1 ],
 [ 0,  4, 53,  0,   0, 80,   0,  4,  5,   0,  0, 0 ],
 [ 4, 660, 3,  0,  30,  0,   3,  0,  4,  90,  0, 0],
 [ 7,  0, 18,  6, 770, 330,  7,  0,  0,   6, 0, 0],
 [ 0, 20,  0,  4,  52,  3,   0,  0,  0,   5, 4, 0],
 [ 0,  0, 40, 70,   4,  63,  0,  0, 60,   0, 4, 0],
 [ 0, 32,  0,  0,   0,  5,   0,  3,  0, 660, 0, 9] ];
```

For this data set, we can get its solution as follows by ILOG planner cooperated with ILOG Solver.

Relaxed cost is 261972, and final Cost is 261922.

Solution: x[j] = [[0] [0] [0] [154] [0] [0] [0] [913] [333] [0] [6499] [1180]]

A comparison of two approaches to get the same result is shown in table 1. It shows that when posting constraints to both CP and LP, the response time is shorter and the memory used is smaller, because the number of fails during the search has been reduced greatly.

6.Remarks

Linear Programming algorithms are widely used in solving linear problems such as liquid blending and production planning. In the paper, a way of linear programming algorithm cooperated with constraint programming is suggested to solve linear integer problems more quickly.

As an optimization tool, ILOG Planner is particularly well suited to standard linear programming problems, while ILOG Solver is based on object-oriented constraint programming. Thus, linear programming algorithm cooperated with constraint programming can be easily achieved by ILOG planner cooperated with ILOG Solver. ILOG Planner can be used to handle linear constraints while ILOG Solver can be used to handle constraint propagation. Besides, this approach can also be used to solve problems for which linear programming techniques are not applicable but involve a lot of linear constraints. By this way, non-linear constraints can also be handled and integrated together. For example, one can add logical constraints, like "If this variable is not zero, these other variables must be zero". When looking for solutions, ILOG Planner relies on the search procedures of ILOG Solver. It benefits from all the flexibility of the ILOG Solver functions in driving the search, implementing search strategies and solving directly logical constraints.

The combination of constraint programming with linear programming can be also very effective when solving problems with both real and integer variables and linear constraints. Combining these two techniques provides an more efficient approach that may be applied to a broad range of application problems.

7. Reference

[1] ILOG Optimization Technology White Paper, ILOG, Inc. 1997. (http://www.ilog.co.jp/)

[2] ILOG Solver, User Manual, version 4.0, May, 1997.

[3] ILOG Planner, User Manual, version 2.0, June, 1997.

Table 1: A comparison of two approaches.

Evaluations items	Post to LP only	Post to both LP and CP
Number of fails	1807	52
Number of choice points	2125	58
Total memory used (bytes)	285636	80616
Elapsed time since creation (seconds)	2.86	0.16