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MIXED-TYPE SECRETARY PROBLEMS ON SEQUENCES OF BIVARIATE RANDOM VARIABLES

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ABSTRACT. An employer interviews a finite number n of applicants for a position. They are interviewed one by one sequentially in random order. As each applicant i is interviewed, two attributes are evaluated by the amounts X_i and Y_i, where X_i may be "talent" (or quality), and Y_i may be the "look" (or degree of favorable impression) of the applicant. Suppose that $\{X_i\}_{i=1}^n$, $(\{Y_i\}_{i=1}^n)$ is under the condition of full (no) -information secretary problem and that X_i's and Y_i's are mutually independent. We consider the three kinds of the employer's object and for each of three cases the problem is formulated by dymanic programming, and the optimal policy is explicitly derived.

1. INTRODUCTION (略)

2. SELECTING GOOD QUALITY UNDER THE REQUIREMENT OF THE BEST LOOK

Let $\{(X_i, Y_i)\}_{i=1}^n\}$ be a sequence of independent bivariate r.v.s as given in the previous section. Observing the sequence (X_i, Y_i), $i = 1, 2, \ldots, n$, one by one sequentially, we want to maximize $\mathrm{E}X_\tau$, where τ is the stopping time, under the required condition that $Y_\tau = 1$ and $2 \le Y_t \le t$, for $\tau < t \le n$.

We define state $(i, x|n)$ which means that: no stop has yet been made, and we face the i-th object whose second attribute is relatively best (i.e. $Y_i = 1$), and the first attribute is found as $X_i = x$.

Denoting, by $V_{n,i}(x)$, the expected reward obtained by employing the optimal stopping rule for the n-object problem at state $(i, x|n)$, we easily have

$$v_{n,i}(x) = \max\{\frac{i}{n}x, \sum_{j=i+1}^{n} \frac{i}{j(j-1)} \mathrm{E}_F v_{n,j}(X)\}, \tag{2.1}$$

$i = 1, 2, \ldots, n$; $v_{n,n}(x) \equiv x$.

Theorem 1. *The optimal stopping rule for the optimal equation (2.1) is : "Stop at the earliest object (X_i, Y_i) whose relative rank Y_i, when it appears, is unity and satisfies $X_i > d_{n,i}$". The sequence $\{d_{n,i}\}_{i=1}^n$ is determined by the recursion*

$$d_{n,i} = \frac{S_F(d_{n,i+1})}{i} + d_{n,i+1}, \tag{2.3}$$

$(i = 1, 2, \ldots, n-1;\ d_{n,n} = 0)$ where $S_F(t) \equiv \mathrm{E}_F(X \vee t)$.

The optimal expected reward is given by $\mathrm{E}_F v_{n,1}(X)$, i.e. $\frac{S_F(d_{n,1})}{n}$.

3. SELECTING GOOD QUALITY UNDER THE REQUIREMENT OF ONE OF THE TWO BEST IN THE LOOK. (略)

4. SELECTING GOOD LOOK UNDER THE REQUIREMENT OF THE BEST QUALITY

Let $\{X_i\}_{i=1}^n$ be an *iid* sequence of r.v.s obeying uniform distribution over the unit interval $0 \le x \le 1$. Observing the sequence $\{(X_i, Y_i)\}_{i=1}^n$, one by one sequentially, we want to maximize $\sum_{r=y}^n \mathrm{P}(R_\tau = r|Y_\tau = y)a_r$, where τ is the stopping time, and a_r is the reward obtained when the absolute rank R_τ of the second attribute at time τ, is r, under the required condition that $X_\tau = \max_{1\le i\le n} X_i$. So the problem is, by following the notation in the previous section,

$$\mathrm{E}\{a_{R_\tau(Y_1,\dots,Y_n)}; X_\tau = \max_{1\le i\le n} X_i\} \to \max_\tau .$$

We assume that $a_1 \ge a_2 \ge \dots \ge a_n \ge 0$.

Define state $(x, y_i|n)$ to mean that no stop has yet been made, and we face the i-th object (x_i, Y_i) with $Y_i = y_i$ and $X_i = \max(X_1, \cdots, X_i) = x$.

Denoting, by $v_{n,i}(x,y)$, the expected reward obtained by employing the optimal stopping rule for the n-period problem at state $(x, y_i|n)$, we easily have

$$\begin{aligned} v_{n,i}(x,y_i) &= \max\left[\frac{x^{n-i}}{\binom{n}{i}}\sum_{r=y_i}^{n}\binom{r-1}{y_i-1}\binom{n-r}{i-y_i}a_r,\right.\\ &\left.\qquad \sum_{j=i+1}^{n}\frac{x^{j-i-1}}{j}\sum_{w=1}^{j}\int_x^1 v_{n,j}(z,w)dz\right], \end{aligned} \tag{4.2}$$

where $1 \le y_i \le i$, $1 \le i \le n$, $0 \le x \le 1$, with $v_{n,n}(x,y) = a_y$.

We discuss the following two simple cases.

Case 1. $a_1 = 1$, $a_2 = a_3 = \cdots = a_n = 0$.

Let $d_{n,i}$ $(i = 1, 2, \dots, n-1;\ d_{n,n-1} = n^{-1},\ d_{n,n} \equiv 0)$ be a unique root in $[0,1]$ of the equation

$$\frac{1}{i}\sum_{l=1}^{n-i}\frac{x^{-l}-1}{l} = 1. \tag{4.6}$$

For some small n, we find that $d_{2,1} = 1/2$, $d_{3,2} = 1/3$, $d_{3,1} = \frac{1+\sqrt{6}}{5} \fallingdotseq 0.6899$, $d_{4,i} = 1/4$, $\frac{1+\sqrt{8}}{7}$ $(\fallingdotseq 0.5469)$, 0.7755 for $i = 3,2,1$ respectively. $d_{5,i} = 1/5$, $\frac{1+\sqrt{10}}{9}$ $(\fallingdotseq 0.4625)$, 0.6591, 0.8248 for $i = 4,3,2,1$ respectively and so on.

Theorem 3. *The optimal stopping region for the optimality equation (4.2) in* Case 1, *is: "stop at the earliest (X_i, Y_i) that satisfies $Y_i = 1$ and $X_i = \max_{1\le t\le i} X_t > d_{n,i}$, where each $d_{n,i}$ is given by a unique root of the equation 4.6.*

Case 2. $a_1 = a_2 = 1$, $a_3 = a_4 = \cdots = a_n = 0$.

For this case the optimality equation (4.2) becomes

$$\begin{aligned} v_{n,i}(x) &= \max\left\{\frac{2n-i-1}{n(n-1)}x^{n-i}, \Psi_{n,i}(x)\right\} \quad \text{for states } (x, y_i = 1|n) \\ u_{n,i}(x) &= \max\left\{\frac{i(i-1)}{n(n-1)}x^{n-i}, \Psi_{n,i}(x)\right\} \quad \text{for states } (x, y_i = 2|n). \end{aligned} \tag{4.10}$$

where

$$\Psi_{n,i}(x) \equiv \sum_{j=i+1}^{n}\frac{x^{j-i-1}}{j}\int_x^1 (v_{n,j}(z) + u_{n,j}(z))dz,$$

Theorem 4. (略)　文献:

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