Software Availability Analysis Considering Intermittent Use

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1 Introduction

We discuss the software availability modeling when the system is used intermittently. From the viewpoint of users, occurrence of a system failure is recognized when the event that either a software failure occurs when the system is in use or a usage demand occurs when the system is under restoration arises. In this paper, a couple of new measures for software availability assessment are derived; these are called the disappointment probabilities in use and under restoration, respectively [1, 2]. It is supposed that the usage demand of the system occurs randomly and that the user's demand time is also random. The time-dependent behavior of the system alternating between up and down states is described by a Markov process [3]. Then the software reliability growth process, the upward tendency of difficulty in debugging, and the imperfect debugging environment are also considered.

2 Model description

The following assumptions are made for software availability modeling:

A1. The software system is unavailable and starts to be restored as soon as a software failure occurs, and the system cannot operate until the restoration action is complete.

A2. The system is not in use at time point zero. The time to occurrence of a usage demand, X, and the usage period, Y, follow exponential distributions with means $1/\theta$ and $1/\eta$, respectively.

A3. The up time (the time to a software failure), Z_n, and the down time (the restoration time), T_n, follow exponential distributions with means $1/\lambda_n$ and $1/\mu_n$, respectively, where n denotes the cumulative number of corrected faults. λ_n and μ_n are decreasing functions of n.

A4. The restoration action implies the debugging activity; this is performed perfectly with probability a $(0 < a \leq 1)$ and imperfectly with probability $b(= 1 - a)$. One fault is removed from the software system when the debugging activity is perfect, and then the software reliability growth and the upward of difficulty in debugging occur.

A5. The usage demands occurring when the system is restored are canceled.

Consider a stochastic process $\{X(t),\ t \geq 0\}$ whose state space is $(\boldsymbol{W},\ \boldsymbol{U},\ \boldsymbol{R})$ defined as follows:

$\boldsymbol{W} = \{W_n;\ n = 0,\ 1,\ 2,\ \ldots\}$
the system is available but not used,

$\boldsymbol{U} = \{U_n;\ n = 0,\ 1,\ 2,\ \ldots\}$
the system is available and used,

$\boldsymbol{R} = \{R_n;\ n = 0,\ 1,\ 2,\ \ldots\}$
the system is restored due to a software failure.

From assumption A4, when the restoration action has been complete in $\{X(t) = R_n\}$,

$$X(t) = \begin{cases} W_n & \text{(with probability } b) \\ W_{n+1} & \text{(with probability } a). \end{cases} \quad (1)$$

Figure 1 illustrates a sample state transition diagram of $X(t)$.

3 Derivation of measures

The state occupancy probabilities $P_{W_n}(t) \equiv \Pr\{X(t) = W_n\}$, $P_{U_n}(t) \equiv \Pr\{X(t) = U_n\}$, and $P_{R_n}(t) \equiv \Pr\{X(t) = R_n\}$ can be obtained analytically, and it is denoted that

$$\Pr\{X(t) \in \boldsymbol{W}\} \equiv \sum_{n=0}^{\infty} P_{W_n}(t), \quad (2)$$

$$\Pr\{X(t) \in \boldsymbol{U}\} \equiv \sum_{n=0}^{\infty} P_{U_n}(t), \quad (3)$$

$$\Pr\{X(t) \in \boldsymbol{R}\} \equiv \sum_{n=0}^{\infty} P_{R_n}(t), \quad (4)$$

respectively. Then the probabilities that a software failure occurs when the system is used and that a usage

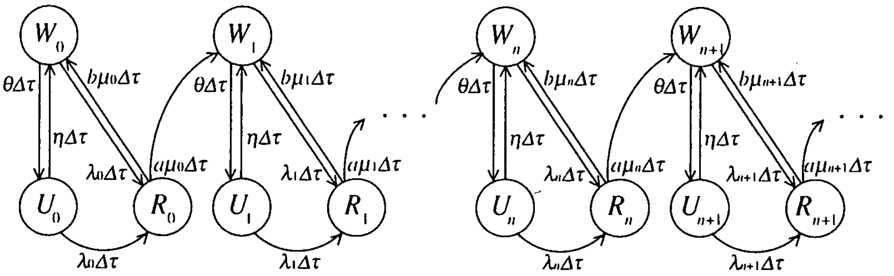

Fig.1 A sample state transition diagram of $X(t)$.

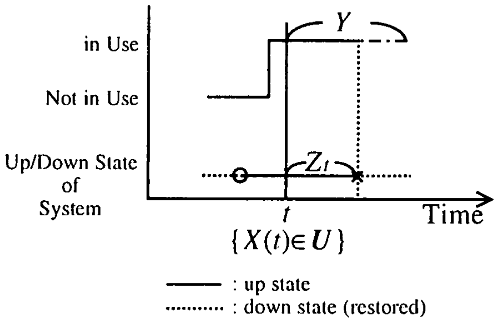

Fig.2 An example of a system failure in use.

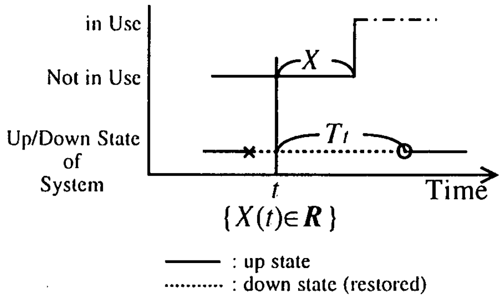

Fig.3 An example of a system failure under restoration.

demand occurs when the system is restored, provided n faults have already been corrected, are given by

$$\Pr\{Z_n < Y\} = \frac{\lambda_n}{\eta + \lambda_n}, \tag{5}$$

$$\Pr\{X < T_n\} = \frac{\theta}{\theta + \mu_n}, \tag{6}$$

respectively.

Let Z_t be the random variable representing the software failure-occurrence time measured from arbitrary time point t. **The disappointment probability in use** is defined as the conditional probability that a software failure occurs during a usage period, provided the system is used at time point t (see Fig.2), and given by

$$H_u(t) \equiv \Pr\{Z_t < Y | X(t) \in \boldsymbol{U}\} = \sum_{n=0}^{\infty} \frac{\lambda_n P_{U_n}(t)}{\eta + \lambda_n} \Big/ \sum_{n=0}^{\infty} P_{U_n}(t). \tag{7}$$

On the other hand, let T_t be the random variable representing the restoration time measured from arbitrary time point t. **The disappointment probability under restoration** is defined as the conditional probability that a usage demand occurs before a restoration action is complete, provided the restoration action is performed at time point t (see Fig.3), and given by

$$H_r(t) \equiv \Pr\{X < T_t | X(t) \in \boldsymbol{R}\} = \sum_{n=0}^{\infty} \frac{\theta P_{R_n}(t)}{\theta + \mu_n} \Big/ \sum_{n=0}^{\infty} P_{R_n}(t). \tag{8}$$

References

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Acknowledgments

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