Dividend and Stock Repurchase Policy with Transaction Costs

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Introduction

Firms mainly distribute cash flows to shareholders in the form of dividends or stock repurchases. Stock repurchase has become an important method of distributing cash flows to shareholders. Firms repurchase stock for the following reasons: to distribute excess cash flow; to announce that firms' managers think their firms' stocks are undervalued; to avoid unwanted takeover attempts; and, to counter the dilution effects of employee and management stock options (Dittmar 2000). In this paper, we examine an optimal dividend and stock repurchase policy with transaction costs under uncertainty. We assume that dividends represent an ongoing commitment, so that the firm pays out dividends in proportion to the accumulated excess cash flow, while, on the other hand, that stock repurchases represent a temporary cash distribution (Jagannathan, Stephens, and Weisbach 2000). In this context, we assume that when the firm pays out dividends it incurs proportional transaction costs, while when it repurchases stock it incurs both fixed and proportional transaction costs.

The Model

We assume that a firm's accumulated net profits are governed by a Brownian motion with drift. $\zeta_t(\in \mathbb{R}_+)$ is the dividend rate at time t and $\xi_i(\in \mathbb{R}_+)$ is the amount of the ith stock repurchase. τ_i is the ith stock repurchasing time. A stock repurchase policy is defined as the following sequence: $v := \{(\tau_i, \xi_i)\}_{i\geq 0}$. Thus, the firm's dividend and stock repurchase policy is defined as the following sequence: $u = (\zeta, v)$. Then, the accumulated excess cash flow $X^{x,u} := (X_t^{x,u})_{t\geq 0}$ is given by

$$X_t^{x,u} = x + \int_0^t (\mu - \zeta_s)ds + \int_0^t \sigma dW_s - \sum_{i=1}^{\infty} \xi_i 1_{\{\tau_i \leq t\}}, \quad 0 \leq t \leq T, \tag{1}$$

where $X_{0-}^{x,u} = x(\in \mathbb{R}_{++})$, $\mu(\in \mathbb{R})$ and $\sigma(\in \mathbb{R}\backslash\{0\})$ are constants, and T represents a bankruptcy time. Let $f : \mathbb{R}_+ \to \mathbb{R}_+$ represent the amount of dividend. Let $K : \mathbb{R}_+ \to \mathbb{R}_+$ represent the amount of stock repurchased. The firm's expected total discounted dividends and stock repurchases function associated with the dividend and stock repurchase policy $u \in \mathcal{U}$ is defined by

$$J(x;u) = \mathbb{E}\left[\int_0^{\infty} e^{-rt} f(\zeta_t)dt + \sum_{i=1}^{\infty} e^{-r\tau_i} K(\xi_i) 1_{\{\tau_i < T\}}\right], \tag{2}$$

$r(\in \mathbb{R}_{++})$ is a discount factor and $\mathcal{U}$ is the set of admissible dividend and stock repurchase policies. Therefore, the firm's problem is to choose $u \in \mathcal{U}$ in order to maximize $J(x;u)$:

$$V(x) = \sup_{u\in\mathcal{U}} J(x;u) = J(x;u^*), \tag{3}$$

where V is the value function of the firm's problem eq. (3) and u^* is an optimal dividend and stock repurchase policy.

Analysis

From that formulation, we naturally guess that, under an optimal dividend and stock repurchase policy, the firm continuously pays out dividends, while the firm repurchases stock whenever the excess cash flow reaches a threshold. In order to verify this conjecture, we take three steps (Theorem 1 –3). To this end, we introduce the following definitions:

Definition 1 (QVI). *Let ϕ be a stochastically C^2 function. The following relations are called the QVI for the firm's problem (3):*

$$\mathcal{L}\phi(x) + f(\zeta) \leq 0, \quad \text{for a.a. } x \text{ w.r.t. } \mathbb{G}(\cdot, x; u),\ u \in \mathcal{U}; \tag{4}$$

$$\phi(x) \geq \mathcal{M}\phi(x); \tag{5}$$

$$\left[\sup_{\zeta \in \mathcal{Z}}[\mathcal{L}\phi(x) + f(\zeta)]\right][\phi(x) - \mathcal{M}\phi(x)] = 0, \quad \text{for a.a. } x \text{ w.r.t. } \mathbb{G}(\cdot, x; u),\ u \in \mathcal{U}, \tag{6}$$

where $\mathcal{L}$ and $\mathcal{M}$ are operators, $\mathbb{G}$ is the Green measure, and $\mathcal{Z}$ is the set of admissible dividend policies.

Definition 2 (QVI policy). *Let ϕ be a solution of the QVI. Then, the following dividend and stock repurchase policy $\hat{u} = (\hat{\zeta}, \hat{v})$ is called a QVI policy:*

$$P\left((t, X_t^{x,\hat{u}}) \in \mathbb{R}_+ \times H; \hat{\zeta}_t \in \arg\sup_{\zeta \in \mathcal{Z}}[\mathcal{L}\phi(X_t^{x,\hat{u}}) + f(\zeta)]\right) = 1; \tag{7}$$

$$(\hat{\tau}_0, \hat{\xi}_0) = (0, 0); \tag{8}$$

$$\hat{\tau}_i = \inf\{t \geq \hat{\tau}_{i-1}; X_t^{x,\hat{u}} \notin H\}; \tag{9}$$

$$\hat{\xi}_i = \arg\max\{\phi(\eta(X_{\hat{\tau}_i-}^{x,\hat{u}}, \xi_i)) + K(\xi_i) : \xi\}. \tag{10}$$

In this context, $\hat{v}$ is defined by $\hat{v} = \{(\hat{\tau}_i, \hat{\xi}_i)\}_{i \geq 0}$ and H is the continuation region defined by $H := \{x; \phi(x) > \mathcal{M}\phi(x) \text{ and } \sup_{\zeta \in \mathcal{Z}}[\mathcal{L}\phi(x) + f(\zeta)] = 0\}$, and $X_t^{x,\hat{u}}$ is the result of applying the dividend and stock repurchase policy $\hat{u}$.

Theorem 1. *We prove that the QVI policy is an optimal dividend and stock repurchase policy for the firm.*

Theorem 2. *We prove that the thresholds (and another unknown parameter) exist and are unique by examining a system of simultaneous equations, which are the well-known value matching and smooth pasting conditions.*

Theorem 3. *We verify that a corresponding candidate function satisfies the QVI under an additional condition, that bounds the threshold level from below. That is, the guessed candidate function is the firm's optimal value function, so that the QVI policy induced by the candidate function is indeed optimal.*

References

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