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Decomposition of Cost Efficiency: Application to Japan-US Electric Utilities Comparisons

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1. Introduction

In this study, we introduce a new framework using the traditional CCR model, and the New Tech and New Cost model in order to decompose the actual cost C_o into the optimal cost C_o^{***} and three types of losses due to input inefficiency to be dependent on technical inefficiency, price inefficiency, and allocative inefficiency[1]. By further extension, we conduct decomposition including scale inefficiency and factor-oriented decomposition. These models are applied to a comparison of electric power utilities in Japan and the US to clarify the main reason for comparatively higher electricity price in Japan.

2. Decomposition of the Actual Cost

Technical efficiency is measured by traditional input-oriented CCR model. The CCR projection to the efficiency frontier is given by

$$[CCR-projection]\ \ x_o^* = \theta^* x_o - s^{-*},\ \ y_o^* = y_o + s^{+*}. \quad (1)$$

Using the input factor price w_{io}, the technically efficient total input cost for DMU$_o$ is calculated as

$$C_o^* = \sum_{i=1}^m w_{io} x_{io}^* = \sum_{i=1}^m w_{io}(\theta^* x_{io} - s^{-*}). \quad (2)$$

The loss in input cost due to this technical inefficiency is expressed as follows:

$$L_o^* = C_o - C_o^* (\geq 0): \textit{Loss due to Technical inefficiency.} \quad (3)$$

Furthermore we apply technically efficient input cost $\bar{x}_{io} = w_{io} x_{io}^*$ to the NTec and NCost Model introduced by Tone (2002). The NTec Projection is given by

$$[NTec-projection]\ \ \bar{x}_o^* = \rho^* \bar{x}_o - t^{-*},\ \ y_o^* = y_o + t^{+*}, \quad (4)$$

and then the price efficient cost $C_o^{**} = \sum_{i=1}^m \bar{x}_{io}^*$ is obtained. Allocative efficient cost C_o^{***} is obtained from the optimal solution of the NCost Model. The losses due to price inefficiency (L_o^{**}) and allocative inefficiency (L_o^{***}) are defined as follows;

$$L_o^{**} = C_o^* - C_o^{**} (\geq 0)\ \ : \textit{Loss due to Price inefficiency,} \quad (5)$$

$$L_o^{***} = C_o^{**} - C_o^{***} (\geq 0): \textit{Loss due to Allocative inefficiency.} (6)$$

Based on the relationship described by EQ(3), (5) and (6), the actual cost (C_o) can be decomposed into three losses and the minimum cost (C_o^{***}):

$$C_o = L_o^* + C_o^* = L_o^* + L_o^{**} + C_o^{**} = L_o^* + L_o^{**} + L_o^{***} + C_o^{***}. \quad (7)$$

3. Extensions of the Model

3.1 The Loss due to Scale Inefficiency

We can measure scale efficiency using the BCC model. The BCC projection is given by

$$[BCC-projection]\ \ x_o^{B*} = \eta^* x_o,\ \ y_o^{B*} = y_o. \quad (8)$$

Generally, scale inefficiency is measured as the gap between the CCR and BCC frontiers. In this paper, we focus on the radial gap; therefore, the BCC projection does not account for slacks[2]. In order to measure scale inefficiency, we use the CCR-projection without considering slacks as follows:

$$[CCR-projection]\ \ x_o^{C*} = \theta^* x_o,\ \ y_o^{C*} = y_o. \quad (9)$$

The cost of the projected x_o^{B*} and x_o^{C*} are obtained as follows:

$$C_o^{B*} = \sum_{i=1}^m w_{io} x_{io}^{B*} = \eta^* \sum_{i=1}^m w_{io} x_{io}, \quad (10)$$

$$C_o^{C*} = \sum_{i=1}^m w_{io} x_{io}^{C*} = \theta^* \sum_{i=1}^m w_{io} x_{io}, \quad (11)$$

$$C_o^{C*} = \theta^* C_o \leq \eta^* C_o = C_o^{B*}. \quad (12)$$

The loss due to scale inefficiency is expressed as the gap between C_o^{B*} and C_o^{C*} as follows:

$$L_o^{s*} = C_o^{B*} - C_o^{C*} (\geq 0): \textit{Loss due to scale inefficiency} (13)$$

We defined the loss due to pure technical inefficiency

[1] Concerning the detailed models, see Tone and Tsutsui (2004).

[2] If the BCC slacks are considered in the BCC-projection, we cannot avoid the case that the BCC-projection is smaller than that of CCR, even though the CCR-projection considers CCR slacks. Furthermore, in the case that we use the slack, considering CCR and BCC projection, we cannot obtain the "radial gap" between the CCR and BCC models.

L_o^{p*} as the remaining part by subtracting L_o^{s*} from L_o^* as follows[3]:

$$L_o^{p*} = L_o^* - L_o^{s*} (\geq 0): \textit{Loss due to Pure Technical inefficiency} \quad (14)$$

Finally, the cost decomposition proceeds as follows:

$$C_o = L_o^{p*} + L_o^{s*} + L_o^{**} + L_o^{***} + C_o^{***}. \quad (15)$$

3.2 Factor-oriented Decomposition

By focusing on each input factor, the decomposition can be examined in greater detail. The losses due to technical inefficiency (L_o^*) and price inefficiency (L_o^{**}) are redefined as follows:

$$L_{io}^* = w_{io}x_{io} - w_{io}(\theta^* x_{io} - s_i^{-*}) = w_{io}(x_{io} - \theta^* x_{io} + s_i^{-*}) \quad (16)$$

$$L_{io}^{**} = \bar{x}_{io} - (\rho^* \bar{x}_{io} - t_i^{-*}), \quad (17)$$

where L_{io}^* and L_{io}^{**} are losses due to the technical and price inefficiencies for input i, respectively, and $L_o^* = \sum_{i=1}^m L_{io}^*$ and $L_o^{**} = \sum_{i=1}^m L_{io}^{**}$. Considering the losses by factor, we can further decompose the actual total cost (C_o) into losses in greater detail than in Eq. (7) as follows:

$$C_o = \sum_{i=1}^m L_{io}^* + \sum_{i=1}^m L_{io}^{**} + L_o^{***} + C_o^{***}. \quad (18)$$

Furthermore, these losses can be classified into factor-oriented losses as follows:

$$C_o = \sum_{i=1}^m L_{io} + L_o^{***} + C_o^{***} \quad (19)$$

where L_{io} is the sum of the technical inefficiency loss (L_{io}^*) and the price inefficiency loss (L_{io}^{**}) for input i. This decomposition is useful for verifying which input factor cost results in a greater loss.

4. Application to Electric Power Utilities

For DMUs, we consider 19 vertical-integrated electric power companies (9 Japanese and 10 US) from 1992 to 1999. Input dataset consists of typical three inputs; capital (divisia index of assets), labor (number of employees) and fuel (BTU). We also use cost data corresponding to each input data. As output data, we use net electricity power sales.

Fig. 1 describes the result of decomposition including scale inefficiency. Although the Japanese average actual

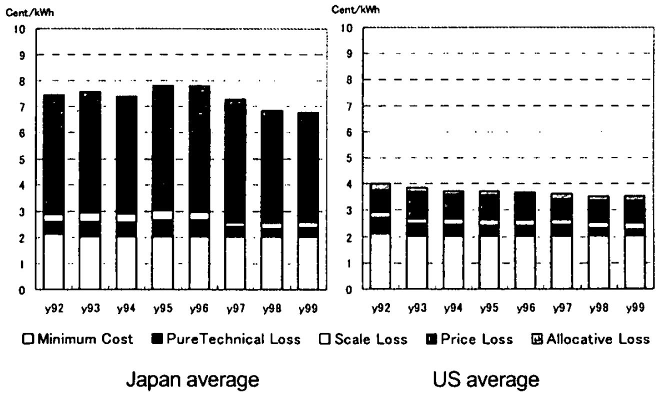

Fig. 1: Further Decomposition including Scale Inefficiency

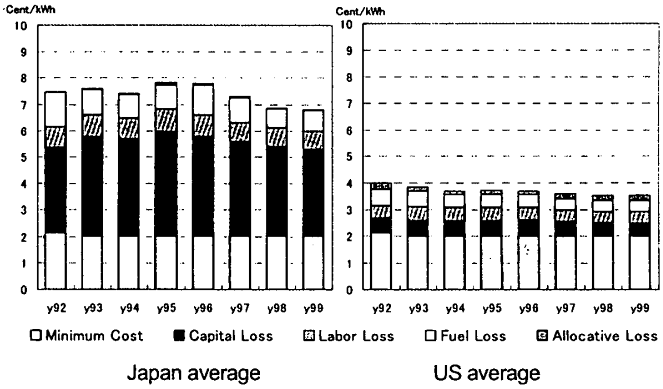

Fig. 2: Factor-Oriented Decomposition

supply cost has been declining over the years, it is nearly twice that of the US average actual supply cost. It is clearly indicated that the losses due to the differences in the input price levels are much larger than those of the others in Japan as well as the losses in the US. In contrast to the price inefficiency loss, there are no significant differences in losses due to pure technical, scale and allocative inefficiencies between Japan and the US.

According to Fig. 2, it is clearly shown that the loss of capital input factor in Japan is much larger than that in the US.

These results imply that the comparatively higher electricity price in Japan is caused by higher input factor prices, not technical inefficiency, and from the factor-oriented point of view, the loss of capital input factor shows large difference between two countries.

Reference

Tone, K. (2002) "A strange case of the cost and allocative efficiencies in DEA," Journal of the Operational Research Society, 53, 1225-1231.

Tone, K., and Tsutsui, M. (2004) "Decomposition of Cost Efficiency and its Application to Japan-US Electric Utilities Comparisons" GRIPS Research Report Series I-2004-0004.

[3] In other words, we define the loss due to pure technical inefficiency including slacks of the CCR Model.