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A Comparison Study of Constraint Programming and Mathematical Programming for Solving Combinatorial Optimization Problems

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1 Introduction

There are many real-world, industrial problems where the possible solutions are so numerous that it is not practical to consider all of them in a search for an optimal feasible solution. These problems are sometimes referred to as combinatorial optimization problems, emphasizing the idea that the number of possible solutions grows combinatorial as the number of decision variables increases. Both constraint programming and Mathematical Programming can be used to model and solve this kind of problems. This paper presents a comparison study of constraint programming and mathematical programming for solving combinatorial optimization problems. It focuses on illustrating the major difference of the two approaches. It also discussed how to build a hybrid cooperating optimizers to provide facilities that can combine these two techniques in order to take advantages of both.

2 Approaches to Combinatorial Optimization Problems

There are many different approaches to combinatorial optimization problems. The best approach depends on the problem as well as the business requirements. In this paper, two optimization techniques are considered. Mathematical Programming, especially, its linear and integer programming are the most well known optimization approaches and are widely used to optimize economic goals. Constraint programming is a relatively new technique that has proved particularly powerful when used to solve scheduling problems and other problems where linear and integer programming are insufficient. These two solution approaches are the main optimization techniques used throughout industry for solving combinatorial optimization problems.

2.1 Constraint Programming (CP)

Constraint Programming was originally designed to solve feasibility problems, but it has been extended to solve combinatorial optimization problems as well. Constraint programming algorithms are very efficient for some classes of combinatorial optimization problems. An advantage of constraint programming in attacking combinatorial optimization problems is that constraint programming offers a variety of modeling facilities, such as logical constraints, higher-order constraints, and global constraints, that make it somewhat easier to formulate a model that is natural and intuitive for people.

These natural, intuitive models make it easier to exploit the problem structure, for example, in global or logical constraints, to find a solution more efficiently. Each time a new search decision is made, all the relevant constraints (linear and nonlinear) are automatically propagated in order to reduce further exploration of the search space.

Constraint programming also facilitates the design of search procedures that exploit the structure of a given problem, and lets users formulate their own goals to guide a search; it also offers predefined evaluators and selectors for controlling the search effectively.

2.2 Mathematical Programming (MP)

Mathematical Programming, especially, its linear and integer programming is a way of applying linear constraints over binary, integer, or real (floating-point) variables. A linear program is made up of a set of linear constraints and a linear objective function. When a linear program is used as a relaxation of a problem it is referred to as a linear relaxation. Generally, a good integer programming solver will maintain an optimal solution of the relaxation of a problem while it generates additional linear constraints, known as cutting planes. Thus, when it is used to attack a combinatorial optimization problem, it is usually emphasized on finding a strong linear relaxation of the problem.

3 A Comparison of Constraint Programming and Mathematical Programming

A comparison of CP-based approaches and MP-based approaches for solving combinatorial optimization problems can be summarized as bellow, in which MP-based approaches refer to approaches which uses Mathematical Programming, especially, its linear and integer programming technology.

3.1 Problem Modeling

Table 1 provides a comparison on problem modeling.

Table 1

		CP-based approach	MP-based approach
Problem modeling	Constraint representation	Any constraints	linear constraints only
	Objective representation	Any expressions	Linear expressions

3.2 Problem Solving

Table 2 provides a comparison on problem solving.

Table 2

	CP-based approach	MP-based approach
Characteristics	Relies on inference	Relies on continuous relaxation
Approaches	From discrete to discrete	From continuous to discrete
Main focus	Constraints and feasibility	Objective function and optimality

Preprocessing	**Pre-solve**	Propagation without search	Pre-solve on root-node, Variable fixing & Cut generation
	Search space deduction	Domain reduction	LP relaxation
Solving	**Search methods**	Goal-based explicit searching with constraint propagation	Built-in branch-and-bound algorithm
	Elimination	Prune: eliminate infeasible domain values	Bound: eliminate suboptimal solutions
	Branching	Domain splitting	Fractional variables, then domain splitting
	Inference during search	Domain reduction and constraint propagation	LP relaxation as lower bound
	Deduce new constraints	Constraint propagation	Strong cutting planes
	Search tuning	User supplies CP search	Parameter tuning

3.3 Advantages and Weaknesses

Table 3 summarized the Advantages and Weaknesses of the two approaches.

Table 3

	Advantages	**Weaknesses**
CP-based approach	Good for finding feasible solutions	Lack of guidance from objective function
	Almost all classes of constraints	Not good for Less constrained problems
	Handle integers directly	May go deep backtracking
	Expressive framework	May slow down with strengthened propagation
MP-based approach	Good at proving optimality	Restricted class of constraints
	Good global reasoning	Weak on handling combinatorial constraints
	Finds optimum bounds without search	May become huge hard MIPs
	Nice theoretical foundation	May slow down when too many integer variables

4 Hybrid Approaches

From the comparison above, we can seen that the two approaches have complementary characteristics. One practical way of attacking combinatorial optimization problems is through constraint programming techniques to reduce the combinatory explosion and Mathematical Programming, especially, its linear and integer programming techniques to provide the bounding of the solutions by utilizing of any linear constraints in the model. Thus, a hybrid approach can be considered to use CP and MP Together.

There are several different approaches for using constraint programming and linear and integer programming together. Some approaches use the optimizers simultaneously; others apply the optimizers sequentially (one after the other). Thus, it is possible to pursue either of the following strategies:

- **Model and Search cooperation:Modeling problems in MP and solving with CP-based search**

The problem can be represented as a integer programming model and can be solved by designing customized branch & bound strategy by means of CP goals. In general terms, this allows to incrementally solve a linear program at each node of a CP search tree and use the information to help the CP search.

- **Double modeling: Deep information exchange while problem solving**

This approach relies on the fact that the linear constraint can manage synchronization with the CP constraints. Thus, hybrid functionalities are available in an instance of linear constraint when it is created with an instance of CP constraints. This can provide an encapsulation linear and integer programming in a global constraint for a problem modeled in CP. An instance of linear constraint will capture all linear constraints that are extracted by the instance of CP constraints or that are added during the search. Because the solution of a linear relaxation is a relaxed optimal solution, this solution helps to guide the search, to tighten variable bounds for better constraint propagation, and to achieve earlier detection of infeasibilities.

- **Problem decomposition: Sequential usage of two approaches**

This approach is to uses instances of CP and MP model sequentially with different "solvers" for each part. In other words, that approach first uses CP to solve a problem; then those solutions are in turn used to create an integer programming problem that can readily be solved by MP model. Column generation can be one of examples of this approach, in which the master problem can be modeled as MP problem while the sub-problem can be modeled as CP.

5 Remarks

This paper presents a comparison study of constraint programming and mathematical programming for solving combinatorial optimization problems. It also presented about hybrid cooperating optimizers that can combine these two techniques and take advantages from both of them.

6 Reference

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[2] ILOG Solver, User Manual, version 6.0, 2004.

[3] ILOG CPLEX, User Manual, version 9.0, 2004.