SALES SIMULATION OF A MOVIE RENTAL SHOP
WITH CUSTOMER AGENTS

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(Received August 13, 2007; Revised March 23, 2009)

Abstract  Customer interaction with a movie rental business can be modeled with a multi-agent simulation consisting of customer, rental shop, and rental transaction models. A customer model is an agent that has independent purchase behaviors. The customer model is described by several properties called customer indicators. A rental shop model is a virtual shop with video stocks and services. The rental shop model is described by several properties called business indicators. A rental transaction model is composed of a series of virtual transactions between customer agents and the rental shop model. The rental transactions are simulated using the day-by-day time-marching method for one year. The results of the simulations are the sales figures of the rental movie shop and the consumption ratios of leisure time and money spent watching rental movies for each customer. Example results demonstrate the dependence of sales results on indicator values, and the potential for these simulations to benefit business management.

Keywords: OR practice, management, mathematical modeling, multi-agent simulation, service modeling

1. Introduction
The ultimate goals of any business include both company revenue and customer satisfaction. These are correlated, as higher revenue can be derived from improved satisfaction, which in turn can result from an improved understanding of customer purchase behavior. This improved understanding can be achieved through simulation that yields estimates of company revenue figures and customer satisfaction levels. An agent is a computer model of an individual person, and a multi-agent system is a model of an artificial society [2, 3]. Studies have applied multi-agent simulations to retail stores [1, 11], and the detailed behavior of store customers was modeled in order to make an ”access and movement” plan for sales staff arrangement or store furniture layout. This approach is only useful for considering agent interaction in the store but cannot be applied to higher management strategies such as pricing. Other studies have modeled customers’ selection behavior, considering attributes from more than one store. These studies demonstrate the potential for retail business administration based on multi-agent simulations. For example, Schenk et al. modeled consumer behavior in grocery shopping on a regional level based on geographic information and multi-agent simulations [10], and Lopez-Sanchez modeled consumer behaviors in on-line music distribution based on consumer segmentation analysis [7–9]. In the service selection model used for [9], the customer selected the least expensive option that passed their threshold for acceptable service. These studies are works in progress and need to address service provider variables and customer attributes explicitly.

This article proposes the application of multi-agent simulation to support business management administration of retail movie stores and demonstrates the results of a prototype
Figure 1: Total concept of rental simulation

simulator. The approach used here is a utility maximization for purchasing model based on multi-agent simulation. Our service selection model deals with both service provider variables and customer attributes explicitly, and calculates each customer’s satisfaction based on service fees and the no-purchase threshold. The prototype simulator was implemented based on information gained from fieldwork at a real rental movie store and data gathered from its customers. The prototype simulator consists of two sub-models: a customer model and a shop model. An interaction between the two sub-models is defined as a rental transaction. The simulations take a time-marching approach, proceeding one day at a time. The results of the simulations include rental revenue of the rental movie stores and individual customer purchasing histories.

2. Developed Approach
2.1. Rental simulation
The purpose of the movie rental sales simulation is to ascertain the most effective strategy for the business to increase customer satisfaction and gain high revenues. A ”rental transaction” is an interaction between a ”customer” and a ”shop,” as shown in Figure 1.

1) If a customer is satisfied with the services of a rental movie shop, they rent a media unit (recorded media in the form of a DVD, Blu-ray disc, videocassette, etc) at the shop, pay a rental fee to the shop, spend leisure time watching the media, and bring the media unit back to the shop.

2) A shop loans media units to customers and receives payment from the customers. The total sum of the payments from customers constitutes the rental revenue of the shop.

3) A rental transaction is the money transfer between a customer and the shop for media units and services.

These three elements are modeled in the simulation system. First, an individual customer is discretely modeled as an agent with individual characteristics. Second, a rental movie shop is modeled as a provider of media units for rent and as a collector of rental fees. Third, the procedures of a rental transaction are modeled as a series of purchase behaviors of a customer when they rent a media unit.

2.2. Customer model
An individual customer is modeled as an independent agent, called a ”customer agent”. The customer agent is generated in two steps.

1) A single customer is modeled by using survey data to construct reasonable indicators of customer characteristics.

2) Distributions of characteristic values across a large number of generated customer agents reflect the overall distributions found from the surveyed data.
Sales Simulation with Customer Agents

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Table 1: Customer indicators; the symbol "*" indicates "the value each month" and the symbol "**" indicates "the value per media unit (videocassette, DVD, Blu-ray disc, and so on)"

<table>
<thead>
<tr>
<th>Customer indicators</th>
<th>Identification number</th>
<th>Maximum leisure budget for rental movies *</th>
<th>Age</th>
<th>Desired number of rental movies *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td></td>
<td></td>
<td></td>
<td>Number of movies rented at once</td>
</tr>
<tr>
<td>Residence</td>
<td></td>
<td></td>
<td></td>
<td>Maximum acceptable rental fee **</td>
</tr>
<tr>
<td>Annual earnings</td>
<td></td>
<td></td>
<td></td>
<td>Maximum acceptable time needed to rent *</td>
</tr>
<tr>
<td>Actual number of rental movies watched *</td>
<td></td>
<td></td>
<td></td>
<td>Preference for popular movies</td>
</tr>
<tr>
<td>Actual leisure time *</td>
<td></td>
<td></td>
<td></td>
<td>Preference for the availability of a wide variety of movie titles</td>
</tr>
<tr>
<td>Actual leisure budget *</td>
<td></td>
<td></td>
<td></td>
<td>Preference for overall level of available copies of each movie title</td>
</tr>
<tr>
<td>Maximum leisure time for rental movies *</td>
<td></td>
<td></td>
<td></td>
<td>1st to 4th favorite categories</td>
</tr>
<tr>
<td>Secondary</td>
<td></td>
<td></td>
<td></td>
<td>Value of leisure time</td>
</tr>
<tr>
<td>Outcome</td>
<td></td>
<td></td>
<td></td>
<td>Cumulative money spent</td>
</tr>
<tr>
<td>Current rental list of movies</td>
<td></td>
<td></td>
<td></td>
<td>Cumulative times spent</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Cumulative rental list of movies</td>
</tr>
</tbody>
</table>

2.2.1. Modeling of a customer

The purchase behavior of any customer can be complex, and a complete understanding of it may necessitate understanding a wide range of values and life situations. For the purposes of this study, however, it is sufficient to focus on attitudes related to renting media units, representing each customer with a set of characteristics called "customer indicators" (Figure 2).

1) Customer indicators

A customer’s movie-watching behavior attributes are categorized into primary, secondary, and outcome indicators. The primary indicators are basic personal attributes, such as age and gender. The secondary indicators, such as value of time, are calculated from the primary indicators. The outcome indicators are the results of the customer agent’s movie watching behaviors, such as time and money spent.

2) Survey

The values of the primary indicators were collected through a survey of actual customers. Surveys composed of 34 questions were randomly sent to more than 300 residents of Matsugasaki, Kashiwa city, Japan in December 2003. Matsugasaki is a typical suburb near a terminal station, as well as a convenient area for data collection. 64 respondents of various ages and backgrounds provided responses to our survey, and 58 respondents were found to be valid and selected for inclusion in the study. From these answers, 26 indicators were selected for customer agents as shown in Table 1. Customer indicator details are included in the appendix.

To analyze the results of the questionnaires, we used the two-step clustering method in Clementine, a well-known commercial software for data analysis. The respondents were clustered into four groups depending on their values for 12 indicators: maximum leisure time...
Table 2: Numbers and average values of the four clusters created by analyzing respondents

<table>
<thead>
<tr>
<th></th>
<th>number of actual customers</th>
<th>maximum acceptable rental fee (yen/unit)</th>
<th>maximum time for rental movies (hours/month)</th>
<th>maximum budget for rental movies (yen/month)</th>
<th>maximum acceptable time needed to rent (hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>heavy users</td>
<td>24</td>
<td>460</td>
<td>9.0</td>
<td>2,104</td>
<td>0.98</td>
</tr>
<tr>
<td>moderate users</td>
<td>12</td>
<td>500</td>
<td>3.1</td>
<td>1,024</td>
<td>0.79</td>
</tr>
<tr>
<td>light users</td>
<td>13</td>
<td>400</td>
<td>1.6</td>
<td>410</td>
<td>0.79</td>
</tr>
<tr>
<td>non-users</td>
<td>9</td>
<td>256</td>
<td>0.13</td>
<td>3</td>
<td>0.22</td>
</tr>
</tbody>
</table>

for watching rental movies each month (hours/month), maximum leisure budget for watching rental movies each month (yen/month), desired number of rental movies each month (media units/month), maximum acceptable rental fee per media unit (yen/media units), maximum acceptable time needed for renting movies (hours), preference for popular movies, preference for the availability of a wide variety of movie titles at the rental shop, preference for overall level of available copies of each movie title in stock at the rental shop, and four favorite categories of movies. During trials of clustering, it was observed that clustering by including indicators of favorite categories yielded a lot of trivial small clusters that were inappropriate for use, so it was decided not to cluster by favorite category. In the process of clustering, the other indicator values were analyzed. Finally, the respondents were classified into four clusters. The four clusters were labeled as heavy users, moderate users, light users, and non-users [5]. Table 2 shows the results of clustering. The distribution of respondent values for each customer indicator is cluster-dependent. Though all the distributions are not strictly normal (Gaussian), it is assumed that each customer indicator has a normal distribution with an average value and standard deviation. The number of respondents may be small but the results we obtained reflect actual behaviors of rental movie customers. These results were used in generating customer agents, as detailed in the following section.

2.2.2. Generation of customer agents

A diverse set of customer agents was generated from the small number of real customers surveyed as follows. One real customer was picked at random from the 58 survey respondents. The selected respondent’s answers were regarded as an original data set for a new customer agent, which was generated by the Monte Carlo method as described below. The cluster that the respondent belongs to was \( i \) (being heavy, moderate, light, or non users), the respondent’s value for primary indicator \( j \) was \( x_j \). The average of customer indicator value was \( m_{j,i} \) and the standard deviation was \( s_{j,i} \) for the customer indicator \( j \) in cluster \( i \). To generate a wide variety of customer agents from 58 respondents, the standard deviation was decomposed into the original value \( s_{1,j,i} \) and the random value \( s_{2,j,i} \).

\[
\begin{align*}
\textstyle s_{j,i}^2 &= s_{1,j,i}^2 + s_{2,j,i}^2 \\
\textstyle s_{1,j,i}^2 &= k_1 \times s_{j,i}^2 \\
\textstyle s_{2,j,i}^2 &= k_2 \times s_{j,i}^2 \\
\textstyle k_1 + k_2 &= 1
\end{align*}
\]

In this article, the values of \( k_1 \) and \( k_2 \) are tentatively set to 0.75 and 0.25, respectively. The value \( x_j \) of the primary indicator \( j \) was converted into the random value \( \hat{x}_j \), while keeping the average and standard deviation unchanged. The lower percentile \( p\{x_j\} \) is described as cumulative distribution function \( F\{x_j|m, s\} \).
\begin{align}
p(x_j) = F\{x_j|m,s_{j,i}\} = \frac{1}{\sqrt{2\pi s_{j,i}}} \int_{-\infty}^{x_j} \exp\left(-\frac{(x - m_{j,i})^2}{2s_{j,i}^2}\right) dx \tag{2.5}
\end{align}

where
- $\hat{x}_{j1}$ is the component of $\hat{x}_j$ based on the decomposed $s_{1,j,i}$, and is determined by the inverse function of cumulative distribution.
- $\hat{x}_{j2}$ is the component of $\hat{x}_j$ and is determined randomly according to a normal distribution with an average value of zero and the standard deviation $s_{2,j,i}$.
- $\hat{x}_j$ is the sum of the value of $\hat{x}_{j1}$ and $\hat{x}_{j2}$.

\begin{align}
\hat{x}_{j1} &= F^{-1}\{x_j|m,s_{1,j,i}\} = \{x : F\{x_j|m,s_{1,j,i}\}\} = p(x_j) \tag{2.6} \\
\hat{x}_j &= \hat{x}_{j1} + \hat{x}_{j2} \tag{2.7}
\end{align}

The customer indicator values are discretely defined, and include preference for popular movies, preference for the availability of a wide variety of movie titles at the rental shop, and preference for the overall level of available copies in stock of each movie title at the rental shop. The values of those indicators were rounded within the defined scope. For example, the customer indicator value for gender of each customer agent was determined based on the selected respondent’s gender and a likelihood of $k_1$ and $k_2$. This procedure for determining gender resulted in rates of each gender in each cluster of generated customer agents that were almost the same as the rates among the surveyed real customers.

The indicator values for favorite categories were determined by the original respondent’s favorite categories at a likelihood of $k_1$ and by the likelihood arising from the multiplication of $k_2$ and the rates of the categories in the cluster to which the original respondent belongs. A customer agent was generated after all indicator values were defined. This is the Monte Carlo method of generating diverse and realistic customer agents. The generation of customer agents was repeated until the number of customer agents needed for simulations was achieved.

The generated customer agents were verified by real customer data in two steps. In the first step, all distributions of the customer indicator values of the generated customer agents were checked and compared with the originals. The correlation coefficient was calculated between the distribution of the original 58 customers and the distribution of 10,000 generated customer agents. In the second step, the correlation coefficient values between one primary indicator and another were checked. The generated customer agents were found to be sufficiently correlated with real data, and were thus able to represent actual customers. The process of modeling a customer agent is shown in Figure 3.

2.3. Shop model

2.3.1. Layer of the shop model

This paper’s movie rental shop models consist of three layered components: a narrowly defined “shop model” (top level), a “category space model” (middle level), and a “media title model” (bottom level). The category space model is a sub-model of the narrowly defined shop model. The media title model is a sub-model of the category space model. The media title models are classified and stored in eight category space models. The eight categories of media are action, science-fiction, horror, mystery, comedy, young adult, romance, and drama. The conceptual diagram of a shop model is shown in Figure 4.
2.3.2. Sub-models of a shop model

The shop model has a set of business indicators similar to those of a customer agent. Each sub-model has two kinds of business indicators: a type detailing the services provided by the shop called "shop service indicators" and a type detailing the results achieved by the shop called "shop outcome indicators". Table 3 shows all the business indicators of the shop model. The details of the business indicators are included in the appendix.

2.3.3. Fieldwork with a real video shop

To survey actual values of shop service indicators, fieldwork at the Kashiwa branch of the rental movie shop TSUTAYA was performed. TSUTAYA is the largest movie rental company in Japan, with more than 1,100 branch shops, 18 million members, and 130 million customers each year. The surveyed branch is a large-scale shop. From fieldwork, the rates of the eight categories of action, science-fiction, horror, mystery, comedy, young adult, romance, and drama are 0.205, 0.132, 0.064, 0.128, 0.125, 0.088, 0.098, and 0.160, respectively. These rates for the eight categories are used in the shop model.

Figure 5 shows the box office revenues of the top 31 movie titles in Japan in 2003 and the number of copies of these titles in stock at the TSUTAYA Kashiwa branch shop in January 2004. The two distributions have a high correlation coefficient of 0.84. Therefore the number of copies of a specific media title in stock was judged to be strongly related to its popularity. In this paper, the number of copies in stock is assumed to be solely determined by the box office revenue. The distribution of the number of copies of specific media titles in stock is exponential decay, approximated by a decay curve with an index coefficient value
Table 3: Business indicators of a shop model; the symbol "*" indicates "the value per media unit" and the symbol "**" indicates "the value each month"

<table>
<thead>
<tr>
<th>Shop service indicator</th>
<th>Category space</th>
<th>Media title</th>
</tr>
</thead>
<tbody>
<tr>
<td>location</td>
<td>category name</td>
<td>title name</td>
</tr>
<tr>
<td>average rental fee *</td>
<td>number of unique movie titles</td>
<td>popularity ranking</td>
</tr>
<tr>
<td>number of unique movie titles</td>
<td>list of movie titles</td>
<td>screen time</td>
</tr>
<tr>
<td>overall level of available copies in stock of each movie title number of copies in stock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>categories</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shop outcome indicator</th>
<th>Category space</th>
<th>Media title</th>
</tr>
</thead>
<tbody>
<tr>
<td>list of media units currently rented out</td>
<td>list of media units currently rented out</td>
<td>available number of the same title in stock</td>
</tr>
<tr>
<td>cumulative list of media units rented out</td>
<td>cumulative list of media units rented out</td>
<td></td>
</tr>
<tr>
<td>rental revenue **</td>
<td></td>
<td></td>
</tr>
<tr>
<td>rental volume of media units **</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5: Relationship between box-office revenues and number of copies in stock

The number of copies of each title in stock is determined as follows. First, each movie title has its own popularity ranking. Second, the number of copies of each title in stock is determined by Equation (2.8).

\[ n_i = n_1 \times f \times e^{\alpha \times (i-1)} \]  

where \( i \) is the \( i \)-th popularity ranking order in the all media titles, \( n_i \) is the number of copies in stock of the movie title, \( n_1 \) is the number of copies in stock of the most popular title, \( f \) is the coefficient for the number of copies in stock indicator of the media title model, and the parameter \( \alpha \) is the decay parameter, having a negative value. In this article, the value of \( n_1 \) is set at 56 units and the value of \( \alpha \) is set at \(-0.02\), as obtained during fieldwork. The shop’s overall level of available copies in stock of each movie title is categorized into seven grades: worst, very poor, poor, ordinary, good, very good and excellent, and the values of the coefficient \( f \) are 0.0, 0.17, 0.33, 0.5, 0.66, 0.83 and 1.0, respectively. The surveyed shop was large-scaled and the value of \( n_1 \), set to 56, represents an excellent grade. The value of \( n_i \) is set as a rounded positive integer, and is set to 1 if the value is less than 0.5.

2.3.4. Shop model generation

The procedures for generating a shop model are described below.
1) Input of basic shop data
The values of the average rental fee of a media unit, the total number of unique movie
titles (counts only the first copy of each unique media title), and overall level of available
copies of each movie title are determined and entered as input by a simulator user. In
the simulator, the eight categories are those described in in 2.3.1.

2) Generating media title models
Every media title model is generated with the order of media unit popularity rankings
going from first to last, so the ranking of the final, least popular media unit is equal
to the total number of unique movie titles in the shop. Every generated media title is
automatically given a unique title name from popularity rankings, and the screen time is
assumed equal to two hours. The number of copies of each generated media title is equal
to \( n_i \). The category for each generated media title model is stochastically determined
in proportion to the ratios of the eight categories.

2.4. Rental transaction model
A customer agent’s selecting and purchasing behaviors are dealt with in the rental trans-
action model. A customer agent compares their own customer indicator values with the
shop service indicators of the shop. The customer agent decides to rent movies when the
shop service indicator values exceed the customer agents’ acceptable levels. Thus, a rental
transaction model consists of four phases:
- evaluation of shop service indicators
- selection of the best shop
- selection of the media title
- making a rental

2.4.1. Evaluation of shop services
The customer agents individually judge the attractiveness of the shops by comparing the
values of their own customer indicators with each shop service indicator values. If there
are several shops available and a customer agent can visit all of them, the customer agent
evaluates them all against their disincentive index. The shop with the lowest disincentive
index after all available shops have been evaluated will receive that customer’s business,
assuming that the customer’s threshold for acceptable service has been met by the shop.
The disincentive index is calculated by customer agents using four shop service indicators:
rental fee, time needed for video entertainment (including geographical proximity of the
customer agent and the shop model), total number of unique movie titles at a shop, and
overall level of available copies in stock of each movie title. The value of the disincentive
index is calculated using Equation (2.9).

\[
\begin{align*}
    r_{di} &= k_1 \times r_{fee} + k_2 \times t_v \times t_e + k_3 \times f_l + k_4 \times f_s \\
\end{align*}
\] (2.9)

where \( r_{di} \) is the value of a customer agent’s disincentive index; \( r_{fee} \) (yen/unit) denotes the
average rental fee per media unit at the shop; \( t_v \) (yen/hour) indicates the customer agent’s
value of time, a secondary indicator; \( t_e \) (hours) is time needed for video entertainment
including screen time of the rented media units and time needed for a rent from home to
the shop; \( f_l \) indicates the customer agent’s satisfaction level, calculated from their primary
indicator for their preference for the availability of a wide variety of movies and the shop
service indicator of the total number of unique movie titles; \( f_s \) indicates the customer agent’s
satisfaction level calculated from their primary indicator of their preference for overall level
of available copies in stock of each movie title and the shop service indicator for overall level
Table 4: Calculation of satisfaction values from a customer agent’s indicators and the shop service indicators

<table>
<thead>
<tr>
<th>Service level of the shop’s offering</th>
<th>Preference level of a customer agent</th>
<th>grade</th>
<th>very high</th>
<th>high</th>
<th>slightly high</th>
<th>ordinary</th>
<th>slightly low</th>
<th>low</th>
<th>very low</th>
</tr>
</thead>
<tbody>
<tr>
<td>worst</td>
<td>very poor</td>
<td>0.00</td>
<td>0.17</td>
<td>0.33</td>
<td>0.50</td>
<td>0.67</td>
<td>0.83</td>
<td>1.00</td>
<td>1.17</td>
</tr>
<tr>
<td>poor</td>
<td>very poor</td>
<td>0.17</td>
<td>0.33</td>
<td>0.50</td>
<td>0.67</td>
<td>0.83</td>
<td>1.00</td>
<td>1.17</td>
<td>1.33</td>
</tr>
<tr>
<td>ordinary</td>
<td>very poor</td>
<td>0.33</td>
<td>0.50</td>
<td>0.67</td>
<td>0.83</td>
<td>1.00</td>
<td>1.17</td>
<td>1.33</td>
<td>1.50</td>
</tr>
<tr>
<td>good</td>
<td>very poor</td>
<td>0.50</td>
<td>0.67</td>
<td>0.83</td>
<td>1.00</td>
<td>1.17</td>
<td>1.33</td>
<td>1.50</td>
<td>1.67</td>
</tr>
<tr>
<td>very good</td>
<td>very poor</td>
<td>0.67</td>
<td>0.83</td>
<td>1.00</td>
<td>1.17</td>
<td>1.33</td>
<td>1.50</td>
<td>1.67</td>
<td>1.83</td>
</tr>
<tr>
<td>excellent</td>
<td>very poor</td>
<td>0.83</td>
<td>1.00</td>
<td>1.17</td>
<td>1.33</td>
<td>1.50</td>
<td>1.67</td>
<td>1.83</td>
<td>2.00</td>
</tr>
<tr>
<td>1.00</td>
<td>1.17</td>
<td>1.33</td>
<td>1.50</td>
<td>1.67</td>
<td>1.83</td>
<td>2.00</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

of available copies in stock of each movie title. In this paper, the stock parameters, \( n_1 \) and \( \alpha \) in the Equation (2.8) in 2.3 are used and specific values were set based on the survey. These specific values can be changed if needed. The satisfaction level of a customer agent is calculated in Table 4 from both customer indicator values and shop service indicators.

The analytic hierarchy process (AHP) was used to determine the weight of these four factors. The survey in subsection 2.2.1 included questions on priority of the factors, making the results amenable to AHP analysis. The AHP involves making one-to-one comparisons between each possible pair in a matrix of options. Mathematical operations in this matrix weigh each option and determine a consistency ratio. Three of the 58 valid respondents had a few missing values in their one-to-one comparison answers, so their pair-wise comparison matrices were incomplete. The missing values in the three incomplete matrices were filled by Harker’s method [4]. AHP should be used only when the value of the consistency ratio is less than or equal to 0.1. 43 of the 58 responses are valid for AHP by this metric. However, all the answers including the inappropriate 15 respondents that have greater than 0.1 values in consistency ratio were used in our analysis. The parameters \( k_1 \) to \( k_4 \) are the sensitivity coefficients of each factor. The weights of the four factors (rental fee, preparation time, title line-up richness, and stock richness of popular titles) were calculated via AHP analysis for each survey respondent. The average weights of the four factors from 58 respondents were 0.21, 0.17, 0.32, and 0.29, respectively. The distribution of weights is depicted in Figure 6.

The standard values of the four factors, average rental fee, time value, \( f_1 \) and \( f_s \) are 350 yen, 401.5 yen/hour \times 2 hours, 1 and 1, respectively.

\[
(k_1 \times 350) : (k_2 \times 401.5 \times 2) : (k_3 \times 1) : (k_4 \times 1) = 0.21 : 0.17 : 0.32 : 0.29 \quad (2.10)
\]

After normalizing the value of \( k_1 \), the final coefficient values of \( k_1 \) to \( k_4 \) are 1.00, 0.350, 527 and 485, respectively.

### 2.4.2. Shop selection

Depending on the values of the disincentive index of the shops, each and every customer agent either visits one shop or none at all. One specific shop is selected if the following three conditions are fulfilled. The first condition is that the ”rental fee” of the shop is lower than or equal to the ”maximum acceptable rental fee” of the customer agent. The second condition is that the ”preparation time for rental” is lower than or equal to the ”maximum acceptable preparation time” of the customer agent. The third condition is that the value of the disincentive index is lower than or equal to the threshold for acceptance of the services. From the shops that fulfill these three conditions, each customer agent selects the shop that has the lowest value of the disincentive index. The threshold value for service acceptance, 1368, is determined by average rental revenue through rental transaction simulations based on the actual data [6].

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2.4.3. Media title selection

The customer agent selects a media title in three steps. For simplicity, all customer agents are assumed not to have a specific title in mind.

step 1: Each customer agent checks the residual values of their leisure time and movie watching budget. If either time or money is not sufficient, the customer agent does not proceed with the rental transaction.

step 2: The customer agent goes to the selected shop model (as described in 2.4.2) depending on the value of their frequency indicator, or rental movie probability (rental times each month).

step 3: The customer agent selects media titles according to their customer indicator values for favorite categories and their preference for popular movies. Each customer agent has a cumulative rental list, so that they do not select media titles that they have already watched. If all selected titles are currently rented out by others, the customer agent selects none and fails to complete a rental transaction.

2.4.4. Making a rental

After passing the described three phases, the customer agent purchases the use of the rental movie, pays rental fees to the shop, deducts the time and money spent from their appropriate budgets for rental movies, and continues spending their remaining budgeted time and money over the course of further simulation. At the same time, the shop gains in rental revenue. Thus, the customer agents spend their time and budget for rental movie services from the shop model by using the rental transaction model. The detail model of the rental transaction procedure is shown in Figure 7.

3. Implementation

All models described in Section 2 were written in JAVA on a personal computer. Figure 8 shows the flowchart of the simulation.

The process of simulation consists of five phases:
Figure 7: Flowchart of the rental transaction model

1) Input of initial data
2) Generation of customer agents
3) Generation of shop models
4) Trading between customer agents and the shops
5) Output of the results of the rental transactions

In the fourth phase, the simulator repeatedly runs the rental transaction model day by day. The one-day simulation will run 365 times for a year-long simulation. A one-day simulation includes all rental transactions between customer agents and shops in one day.

4. Application Results and Discussions

Two applications of our simulation method are shown in this section. From the simulation results, we can obtain an estimate of a shop’s rental revenue and the values of the consumption ratios for time and budget spent by all customer agents under certain conditions.

4.1. Conditions of simulations

Table 5 shows data from the real movie rental shop [6] and a shop model. Some values of shop service indicators in the shop model were determined based on real data. For simplicity, three assumptions are introduced in the simulations. First, a rental shop does not replace old media titles with new releases, and no customer agent rents media units that they have seen before. Second, all customer agents rent media units for two days and return them to the shop on the third day without exception. Third, the screen time of every media title is two hours without exception. The simulations were carried out for thirteen months, and the results of the first month were neglected. The purpose of the first month of simulations is to reach a steady state.
4.2. Results of simulations

The results of the simulations were automatically logged as outcome indicators for all customer agents and shops.

1) Results related to the customer agent
   - money spent
   - time spent
   - cumulative list of rented movie titles
   - budget consumption ratio: ratio of money spent to the overall budget for movie rentals
   - leisure time consumption ratio: ratio of time spent to the planned leisure time for movie rentals

Table 5: Average values from real rental shops [6] and values of the shop model

<table>
<thead>
<tr>
<th></th>
<th>actual shop</th>
<th>virtual shop</th>
</tr>
</thead>
<tbody>
<tr>
<td>members</td>
<td>9,228</td>
<td>10,000</td>
</tr>
<tr>
<td>rental fee</td>
<td>334 yen</td>
<td>350 yen</td>
</tr>
<tr>
<td>number of unique media titles</td>
<td>16,111</td>
<td>16,000</td>
</tr>
<tr>
<td>total media units (counting copies)</td>
<td>20,030</td>
<td>18,677</td>
</tr>
<tr>
<td>monthly rental revenue</td>
<td>4.65 million yen</td>
<td>–</td>
</tr>
<tr>
<td>monthly rental volume of media units</td>
<td>13,931</td>
<td>–</td>
</tr>
</tbody>
</table>
2) Results related to the shop
   - rental revenue
   - rental volume of media units
   - cumulative list of rented movie titles

Desirable rental transactions have higher values for consumption ratios of time and budget
for all customer agents, and for rental revenue of the shop.

4.3. Pricing simulations

The effects of pricing on the volume of rentals and customer satisfaction were studied. In
these simulations, only one shop was present. The results of the pricing effect are shown in
Figures 9a and b. The rental shop’s prices had a stepwise increase of 50 yen from 100 yen
to 600 yen and the stock level varied from excellent (level 1) to worst (level 7) via ordinary
(level 4). The other values of the shop service indicators in the shop model remained fixed.

Figure 9a shows that the rentals of media units decline as the rental fee increases, and
that the distribution of the rental revenue peaks at 300 yen. The reason why the distribution
has its peak there is that at that point there is a good balance between the rental fee income
from each media unit rented out and the volume of rentals from customers who accept the
rental fee. The rental revenue is equal to the multiplication of the rental fee and the rental
volume, and thus it peaks when the rental fee has a certain value. The rental fee for the
highest rental revenue is 300 yen at the ordinary stock level. In almost all of the rental
video shops in Japan, the unit movie rental fee tends to be between 300-350 yen according
to some surveys [6, 12]. The results indicate that 300 yen is the optimum rental fee with the
largest rental revenue when no competing shop is present (as is the case in our simplified
simulations). As stock levels decline, the rental fee for peak revenue drops. For bad stock
levels, the rental fee for peak rental revenue is 250 yen, while it is 300 yen for both ordinary
and excellent stocks. In Figure 9a it is shown that rental revenue for stores with excellent
stock levels do not drop off as sharply compared to stores having only ordinary stock levels
when the rental fee is more than 500 yen. The vertical bars of Figure 9a show rental revenue
(million yen) and the plotted lines show the rental volume of media units. The values are the
monthly averages of 100 trials of one-year-long simulations. The budget consumption ratio
results for each stock level in Figure 9b show a similar distribution as the results of the rental
revenue for each stock level in Figure 9a. The reason for this similarity is that the total
rental revenue of the shop is equal to the sum of all money spent by all the customer agents
when only one shop exists. The dotted-lines of Figure 9b indicate leisure time consumption
ratios (%), and the solid-lines indicate budget consumption ratios (%). The values are the
monthly averages of 100 trials of one-year-long simulations. The leisure time consumption
ratio results for each stock level in Figure 9b are also similar to the sales results for each
stock level in Figure 9a. This similarity is present because the total rental volume of media
units is related to the total time spent by all customer agents.

Let us suppose that a shop changes the stock level of copies for each media title from the
ordinary level to the excellent level, while the shop’s rental fee remains 350 yen. Figure 9
suggests the rental revenue will rise from 4.79 million yen per month to 4.99. Furthermore,
the shop can keep the same rental revenue of 4.79 million yen per month after raising the
rental fee to 365 yen, which can be seen from the results of additional fee simulation. Extra
costs for upgrading stocks are ignored in these simulations.

4.4. Case of competition between two shops

In the next example, two competing shops exist in the same market area. The effects of
pricing are studied when there is competition between two shops. Shop 1 is the target shop
Figure 9: Graph (a) shows the effects of pricing on the sales of the shop, and graph (b) shows the effects of pricing on the consumption ratios of customer agents.

with variable rental fees and Shop 2 is the competitor with all shop service indicators fixed. The values of shop service indicators for each shop are shown in Table 6. Shop 1 is more favorable for all customer agents than Shop 2 in terms of total number of unique movie titles (Shop 1: 18,000 titles, Shop 2: 16,000 titles) and overall level of available copies in stock of each title (Shop 1: good (level 3), Shop 2: ordinary (level 4)) and less favorable in terms of location (Shop 1 is 30 minutes away from Shop 2). For simplicity, the convenience of a shop’s location is made the same for all customer agents. In this simulation, the rental fee in Shop 1 was increased stepwise by 50 yen, from 300 yen to 600 yen, while the rental fee in Shop 2 remained fixed at 350 yen.

The results are shown in Figure 10. The most notable feature of Figure 10a is that the total rental revenue of all shops has a peak in the competition case similar to the case where only one shop is present. Under the conditions of Table 6, the point at which the customer agents as a whole spend the most money from their budgets occurs when Shop 1’s rental fee is set at 450 yen. Demanding customer agents whose time value is low and expectation of high-level service with regards to the two indicators in shop service, total number of unique movie titles and overall level of available copies in stock of each movie title, may tend to prefer Shop 1 rather than Shop 2. These customer agents do not rent movies from Shop 2, because the values of shop service indicators of Shop 2 do not fulfill the three conditions described in 2.4.2. To put it another way, all of Shop 1’s customers who pay a rental fee of 350 yen do not switch to Shop 2 as Shop 1’s rental fee rises. When Shop 1’s rental fees are set at 400 to 500 yen, the money spent by each customer agent who accepts the rental service is relatively higher than other rental fees, and the total money spent is the largest at 450 yen.

Figure fig:result-2shop1a also shows that the total volume of media units rented by Shop 1 and Shop 2 combined decreases as Shop 1’s rental fee increases. Additionally, the rental volume of Shop 1 alone declines as Shop 1’s rental fee increases. Shop 2’s rental fee is constant, so the rental volume of media units in Shop 2 increases as Shop 1’s rental fee increases. Shop 1’s decreasing volume always exceeds the increase in volume at Shop 2’s as Shop 1’s rental fee increases. The bar graphs of Figure fig:result-2shop1a show total rental revenue of Shop 1 and Shop 2 (million yen/month) and the plotted lines show the total rental volumes of media units for Shop 1, Shop 2 and sum of the two shops. The values are
Figure 10: Graph (a) shows the effects of pricing on total sales, and graph (b) shows the effects of Shop 1’s pricing on consumption ratios of all customer agents.

Table 6: Shop service indicators of the two shops

<table>
<thead>
<tr>
<th></th>
<th>Shop 1</th>
<th>Shop 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of unique media titles for rent</td>
<td>18,000</td>
<td>16,000</td>
</tr>
<tr>
<td>stocks level of each movie title</td>
<td>good</td>
<td>ordinary</td>
</tr>
<tr>
<td>rental fee (yen)</td>
<td>300 – 550</td>
<td>350</td>
</tr>
<tr>
<td>time needed for a rental (hours)</td>
<td>+0.5</td>
<td>0</td>
</tr>
</tbody>
</table>

the monthly averages and the averages of 100 trials of one-year-long simulations.

When the rental fee of Shop 1 is less than or equal to 350 yen, Shop 1 completely dominates, with a market share reaching 100%. Shop 1’s rental revenue is the highest when its rental fee is 300 yen. When Shop 1’s rental fee is greater than or equal to 550 yen, it loses its entire market share.

Figure 10b shows that the total budget consumption ratio reaches the highest value when Shop 1’s rental fee is set at 450 yen. The solid line of Figure 10b shows the budget consumption ratios (%), and the plotted line shows the leisure time consumption ratios (%). Figure 10b also shows that the highest value of the customers’ leisure time consumption ratios occurs when Shop 1’s rental fee is set at 250 yen, which is the lowest price in these simulations. When Shop 1’s rental fee is between 400 yen and 500 yen, the total leisure time consumption ratios seem flat. This suggests that the total number of customer agents who accept the rental service is approximately constant when the rental fees are 400-500 yen. The maximum amount of budget spent by customers to rent movies occurs when Shop 1’s rental fee is 450 yen under the conditions of Table 6.

Figure 11 shows the results of an additional simulation series. For these simulations, the number of titles at Shop 1 was increased to 32,000 and the convenience of its location was changed to be the same as Shop 2. All values of shop service indicators of Shop 2 remained the same as in Table 6. Figure 11 shows the rental revenue of Shop 1 against changes in rental fee and stock level for total number of unique movie titles in Shop 1. In the cases of good, very good, and excellent stock levels, Shop 1 was more competitive than Shop 2 even if their rental fee was 500 yen. In the case of an ordinary stock level, Shop 1 was more competitive when the rental fee was 400 yen or less. In the case of poor stock level, Shop 1 was more competitive when the rental fee was 350 yen or less. In the cases of very poor level
and worst stock level, Shop 1 was not competitive even if the rental fee was 300 yen. In these simulations, the extra costs of upgrading stocks were ignored. The simulation results should be validated in the field and we intend to undertake these evaluations in future works.

In the rental movie industry some new emerging services, which for example deliver media units to customers’ home, are developing. Convenient order processes without visiting physical shops and rich stocks are some of the factors causing successes for the delivery business model. In Japan, delivery services and even newer services for on-demand streaming of video in the home have been begun by some service providers. Our simulation methods can support service providers by allowing them to compare newer and traditional services quantitatively.

5. Conclusions

In this article, we discussed the multi-agent approach to simulating individual customer behaviors in renting movies. We studied actual customer behavior with questionnaires and a real movie rental shop through fieldwork and statistics. We implemented a multi-agent simulation for the management of a movie rental shop. We demonstrated two examples of realistic simulations of the business by using a discrete, time-marching method. We focused on pricing in our examples here and will examine how other indicators, such as the selection of media titles available, affect sales and satisfaction ratings in future work. This approach can be extended to study other sales operations.

The ultimate goal of the retail sales business is to establish mutual benefit for customers and service providers. These sales simulations could help find the optimum strategy for maximizing sales of shops and the satisfaction of customers. These discrete and time-marching simulations will be powerful tools to help develop winning sales strategies so that both providers and customers benefit.
Acknowledgment
The authors are indebted to Mr. Lee Rynearson of the Kanazawa Institute of Technology for assistance in translating the manuscript.

References

Appendix
A. Customer Indicators
The customer agents have 26 indicators categorized into primary, secondary, and outcome indicators.

A.1. Primary indicators
- name or identification number
- age
• gender: male or female
• residence: xy-coordinates of home address
• annual income (yen/year)
• actual number of watched rental movies each month (media units/month)
• actual leisure time each month (hours/month)
• actual leisure budget each month (yen/month)
• maximum leisure time to spend watching rental movies each month (hours/month): upper limit of time for monthly watching movies
• maximum leisure budget for watching rental movies each month (yen/month): upper limit of monthly budget for watching movies
• desired number of rental movies each month (media units/month)
• number of movies rented at a time (media units)
• maximum acceptable rental fee per media unit (yen/media units)
• maximum acceptable time needed to rent movies (hours)
• preference for popular movies: tendency to watch “hot” movies quantified as follows: very high (preference for movies ranked higher than 10 in popularity), high (within 30), slightly high (within 60), ordinary (within 100), slightly low (within 150) and low (within 200), and very low (all movies)
• preference for the availability of a wide variety of movies at the rental shop: preference quantified in terms of very high (more than 21,000 titles), high (19,000-21,000 titles), slightly high (17,000-19,000 titles), ordinary (15,000-17,000 titles), slightly low (13,000-15,000 titles), low (11,000-13,000 titles), and very low (less than or equal to 11,000 titles)
• preference for overall level of available copies in stock of each movie title: preference quantified in terms of very high, high, slightly high, ordinary, slightly low, low, and very low. For an example of how the seven levels are found, if the maximum stock level $n_1$ in Equation 2.8 is 56 the seven levels would be as shown below:
  
  - very high: more than 56 media units ($f=1.0$)
  - high: 46 to 55 media units ($f=0.83–1.0$)
  - slightly high: 37 to 45 media units ($f=0.66–0.83$)
  - ordinary: 28 to 36 media units ($f=0.5–0.66$)
  - slightly low: 18 to 27 media units ($f=0.33–0.5$)
  - low: 9 to 17 media units ($f=0.17–0.33$)
  - very low: less than or equal to 8 media units ($f=0–0.17$)

• most favorite movie category: action, science-fiction, horror, mystery, comedy, young adult, romance or drama
• second favorite movie category: action, science-fiction, horror, mystery, comedy, young adult, romance or drama
• third favorite movie category: action, science-fiction, horror, mystery, comedy, young adult, romance or drama
• fourth favorite movie category: action, science-fiction, horror, mystery, comedy, young adult, romance or drama

A.2. Secondary indicators
• value of leisure time (yen/hour): time value calculated from the value of actual leisure budget each month divided by the value of actual leisure time each month
A.3. Outcome indicators
- cumulative money spent on rental movies (yen)
- cumulative time spent on rental movies (hours)
- current list of rental movies: list of currently borrowed media units
- cumulative list of rental movies: list of all previously rented media units

B. Business Indicators of a Shop Model
The shop model consists of three layers: the shop model (narrowly defined), the category space model, and the media title model. Each layer model has two major components: shop service indicators and shop outcome indicators.

B.1. Indicators of a narrowly defined shop model
B.1.1. Shop service indicators
- location: xy-coordinates of address of shop
- average rental fee of a media unit (yen/media unit)
- total number of unique movie titles
- overall level of available copies in stock of each movie title: rated on seven levels including excellent, very good, good, ordinary, poor, very poor and bad. For an example of how to find the seven levels corresponding to each month each month the number of copies of each movie title in stock, if the maximum stock \( n_1 \) in Equation 2.8 is 56, the seven levels would be as described below:
  - excellent: more than 56 media units \((f=1.0)\)
  - very good: 46 to 55 media units \((f=0.83-1.0)\)
  - good: 37 to 45 media units \((f=0.66-0.83)\)
  - ordinary: 28 to 36 media units \((f=0.5-0.66)\)
  - poor: 18 to 27 media units \((f=0.33-0.5)\)
  - very poor: 9 to 17 media units \((f=0.17-0.33)\)
  - bad: less than or equal to 8 media units \((f=0-0.17)\)
- categories: action, science-fiction, horror, mystery, comedy, young adult, romance and drama categories

B.1.2. Shop outcome indicators
- rental revenue (yen): cumulative money earned
- rental volume of media units: cumulative number of media units that were rented in the past
- list of media units currently rented out
- cumulative list of media units rented out

B.2. Category space model
B.2.1. Shop service indicators
- name: category name
- number of media units of unique movie titles in this category
- media units list: list of media titles in order of popularity in a given category

B.2.2. Shop outcome indicators
- list of media units currently rented out
- cumulative list of media units rented out
B.3. Media title model

B.3.1. Shop service indicators
- title name: identification number
- rental fee (yen)
- overall popularity ranking: popularity ranking among all titles at the movie rental shop
- category popularity ranking: popularity ranking in the media’s category at the movie rental shop
- screen time (hours)
- number of copies in stock of the title

B.3.2. Shop outcome indicator
- available number of copies in stock: number of copies in stock of the title minus the number of units of that title that are rented out

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