A SIMPLEX PROCEDURE FOR A FIXED CHARGE PROBLEM

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Abstract An approximate solution method for solving the optimization problem which contains semi-fixed costs represented as a lower semi-continuous step function is developed. The fundamental idea of the algorithm is based on the simplex procedure of linear programming. We define the decrease in the objective function considering twice pivot calculations, and preparing two kinds of simplex tableau we propose the computational procedure to systematically obtain the approximate solution. Also some properties of the pivot calculations are theoretically analyzed. Finally some numerical examples are solved to illustrate the procedure and to test the effectiveness of the algorithm.

1. Introduction

In this paper, we develop an algorithm for solving the optimization problem which contains semi-variable costs represented as a piecewise linear function shown in Figure 1 and semi-fixed costs represented as a lower semi-continuous step function shown in Figure 2.

Fig. 1 Semi-variable costs
Consider the problem of minimizing

\[ \mathcal{P} = \sum_{k=1}^{l} c_k x_k + \sum_{k=1}^{l} d_k (x_k) \]

subject to

\[ \sum_{k=1}^{l} a_{ik} x_k \geq b_i, \quad 0 \leq x_k \leq u \quad (k=1, \ldots, l), \text{ and} \]

\[ \begin{cases} \xi_{jk} \geq 1, \text{ then } \xi_{ik} = 1 & \text{for } i=1, \ldots, j-1 \\ \xi_{jk} = 0, \text{ then } \xi_{ik} = 0 & \text{for } i=j+1, \ldots, n_k \end{cases} \]

where

\[
A_k = \begin{pmatrix} a_{1k} & \cdots & a_{jk} & \cdots & a_{nk} \end{pmatrix}, \quad \begin{pmatrix} \alpha_{1k} & \cdots & \alpha_{jk} & \cdots & \alpha_{nk} \end{pmatrix}, \quad b = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix}, \quad \begin{pmatrix} \gamma_{1k} \\ \vdots \\ \gamma_{nk} \end{pmatrix}, \quad d_k = \begin{pmatrix} \delta_{1k} \\ \vdots \\ \delta_{nk} \end{pmatrix}, \quad u = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}.
\]
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\[ \mathbf{x}_k = \left( \begin{array}{c}
\xi_{1k} \\
\vdots \\
\xi_{nk}
\end{array} \right), \quad [\mathbf{x}_k] = \left( \begin{array}{c}
[\xi_{1k}] \\
\vdots \\
[\xi_{nk}]
\end{array} \right), \quad \text{and} \quad [\xi_{ik}] = \begin{cases} 
1 & \text{if } \xi_{jk} > 0 \quad (j=1,\ldots,n_k; k=1,\ldots,l), \\
0 & \text{if } \xi_{jk} = 0
\end{cases}. \]

The constraint (3) means that variable \( \xi_{1k} \) \( (i=1,\ldots,j-1) \) have to take the value 1 if the variable \( \xi_{jk} \) takes a positive value and \( \xi_{ik} \) \( (i=j+1,\ldots,n_k) \) have to take the value 0 if \( \xi_{jk} \) takes 0. Also it is assumed that it holds

\[
\alpha_{ij} \geq \alpha_{i,j-1k} \quad (i=1,\ldots,m; j=2,\ldots,n_k; k=1,\ldots,l)
\]

\[
\gamma_{jk} \geq \gamma_{j-1k} \quad (j=2,\ldots,n_k; k=1,\ldots,l)
\]

\[
\delta_{jk} \geq \delta_{j-1k} \quad (j=2,\ldots,n_k; k=1,\ldots,l).
\]

The prime represents the transposition of vectors.

Originally this problem appeared when determining the production planning for the mixed-model assembly line production system [1]. In [1], the problem is formulated as a kind of separable programming which minimizing the objective function constructed from the sum of a convex function and a kind of step function under the constraints of linear inequalities. Approximating the convex function as a piecewise linear function and generalizing the problem, we have relations from (1) to (4). The problem of minimizing (1) subject to (2) and (3) is considered to be a kind of the fixed charge problem and, introducing 0-1 variables, we can treat this problem as a mixed-integer programming problem [3]. Also an algorithm which is based upon a branch and bound method is presented for the general fixed charge problem [4].

In this paper, we will attempt to solve the problem (1)-(3) by means of the simplex method. Though some approximate solution methods using the simplex method have been proposed for the fixed charge problem [2,6,7], we will derive an another approximate algorithm from the different point of view making use of

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following properties of the problem, that is,

(a) From the assumption (4), for the problem of minimizing only the first term of the objective function (1) subject to (2) and (3), we can carry out the ordinary calculations of the simplex algorithm without considering the restriction (3) and, in optimal state, the restriction (3) is automatically satisfied [5].

(b) From the restriction (3), for the problem (1)-(3), we know that if the variable \( x_{jk} \) is a basis and it holds \( 0 < x_{jk} < 1 \), then \( x_{j+1k} \) is the only candidate variable which enters into the basis and \( x_{i'k} \) (\( i=j+2,\ldots,n_k \)) must not enter into the basis before \( x_{j+1k} \). Also, if \( 0 < x_{jk} < 1 \), then \( x_{jk} \) is the only candidate variable which moves to the nonbasis and \( x_{i'k} \) (\( i=1,\ldots,j-1 \)) must not move to the nonbasis before \( x_{jk} \).

The algorithm proposed is essentially constructed with two phases.

(a) First, without considering fixed charges, ordinary simplex calculations are carried out to obtain the initial feasible solution.

(b) Next, considering fixed charges, twice pivot calculations method are carried out to search for a better extreme point assuring the feasibility and monotone decreasing.

2. Preparations for the Algorithm

2.1 Definitions of the sets

Introducing slack variables \( y \) and \( z_k \) (\( k=1,\ldots,l \)), we can represent inequalities (2) as follows:

\[
\begin{align*}
\sum_{k=1}^{l} A_{k} x_{k} - y & = b \\
x_{k} + z_{k} & = u \quad (k=1,\ldots,l)
\end{align*}
\]
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(7) \[ x_k, y, z_k \geq 0 \quad (k=1, \ldots, l), \]

where \( y = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_m \end{bmatrix} \) and \( z_k = \begin{bmatrix} \zeta_{1k} \\ \vdots \\ \zeta_{nk} \end{bmatrix} \).

Let define the function \( G \) as follows:

(8) \[ G = \sum_{k=1}^{l} c_k x_k. \]

We will call the linear programming problem of minimizing (8) subject to (5), (6) and (7) Problem A and the fixed charge problem of minimizing (1) subject to (5), (6) and (7) Problem B hereafter.

Let \( X \) be the set of \( \xi_{j,k} \) (\( j=1, \ldots, n \); \( k=1, \ldots, l \)), \( Y \) be the set of \( \eta_i \) (\( i=1, \ldots, m \)) and \( Z \) be the set of \( \zeta_{j,k} \) (\( j=1, \ldots, n \); \( k=1, \ldots, l \)). At the any step of the simplex iteration, we define the sets of variables as follows:

\[ X_B^1 = \{ \xi_{j,k} \mid \xi_{j,k} = 1, \xi_{j,k} \in X \}, \]
\[ X_B^2 = \{ \xi_{j,k} \mid 0 < \xi_{j,k} < 1, \xi_{j,k} \in X \}, \]
\[ X_N = X - (X_B^1 \cup X_B^2), \]
\[ Y_B = \text{the set of basic variables of } \eta_i \in Y, \]
\[ Y_N = Y - Y_B. \]

† If there exists a basic variable which takes \( \xi_{j,k} = 0 \) (that is, in the presence of degeneracy), we replace it by the nonbasic variable \( \eta_i \in Y_N \). This pivot calculation is always attainable. Let proof this fact. Denote the coefficient matrix for the set of nonbasic variable \( Y_N \) as \( Y_N^* \). Since the inverse matrix \( \Lambda^{-1} \) of the coefficient matrix \( \Lambda \) for the basic variable vector \( w_B \) defined as (11) is represented as (12) mentioned in chapter 3, the coefficient matrix \( Y_N^* \) of the current simplex tableau becomes as follows: \( \Lambda \).
2.2 Definitions of the decrease in the objective function

Let denote the variable which enters into the basis as \( \omega_t^1 \in W_N \) and the variable which moves to the nonbasis as \( \omega_s^1 \in W_B \). Then from the theory of linear programming, the variation \( \Delta H_1 \) in the objective function of Problem B becomes as follows:

\[
\Delta H_1 = (\gamma_t^1 - \pi_t^1) \beta_s^1 / \sigma_t^1 + (\delta_t^1 - \delta_s^1),
\]

where \((\gamma_t^1 - \pi_t^1)\) is the simplex criterion of the nonbasic variable \( \omega_t^1 \), \( \beta_s^1 \) is the value of the basic variable \( \omega_s^1 \), and \( \sigma_t^1 \) is the value of the pivot element. The asterisk represents the value of the current simplex tableau. \( \delta_t^1 \) and \( \delta_s^1 \) are

\[
Y^*_N = K^{-1} Y_N = \begin{pmatrix}
0 & 0 & 0 & -E \\
0 & E & 0 & 0 \\
-\delta_t^1 & -\delta_s^1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

Hence we know that there exists at least one nonzero element for \( \eta_t \in Y_N \) corresponding to the basic variable which takes \( \xi_{tj} = 0 \).
fixed charges of $\omega^1_t$ and $\omega^2_s$, respectively. Let assume that, after we replace $\omega^1_s \in W_B$ by $\omega^1_t \in W_B$, we choose the variable which enters into the basis as $\omega^2_t \in W_N - \{\omega^1_s\}$ and the variable which moves to the nonbasis as $\omega^2_s \in W_B - \{\omega^1_t\}$.

Then the variation $\Delta H_2$ in the objective function becomes as follows:

$$\Delta H_2 = \Delta H_1 + \left(\gamma^2_t - \pi^2_t\right) \beta^2_s / \sigma^2_s + \left(\delta^2_t - \delta^2_s\right),$$

where $(\gamma^2_t - \pi^2_t)$, $\beta^2_s$ and $\sigma^2_s$ are defined as similar values as $(\gamma^1_t - \pi^1_t)$, $\beta^1_s$ and $\sigma^1_s$ corresponding to $\omega^2_t$ and $\omega^2_s$. We can easily calculate $\Delta H_1$ and $\Delta H_2$ if $\omega^1_s$, $\omega^1_t$, $\omega^2_s$ and $\omega^2_t$ are determined. Then we define the decrease in the objective function of Problem $B$ as follows:

Definition. We define that the objective function of Problem $B$ decreases if it holds either

(a) $\Delta H_1 < 0$ or
(b) $\Delta H_1 \geq 0$ and $\Delta H_2 < 0$.

3. Meaningful Pivot Calculations

When we choose the variable which enters into the basis as $\omega^1_t \in W_N$ and the variable which moves to the nonbasis as $\omega^1_s \in W_B$ for the first pivot calculation, we can formally consider fifteen cases as the combination of $\omega^1_s$ and $\omega^1_t$, that is,

1. $\omega^1_s \in X^1_B$, $\omega^1_t \in X_N$
2. $\omega^1_s \in X^2_B$, $\omega^1_t \in X_N$
3. $\omega^1_s \in Y_B$, $\omega^1_t \in X_N$
4. $\omega^1_s \in Z^1_B$, $\omega^1_t \in X_N$
5. $\omega^1_s \in Z^2_B$, $\omega^1_t \in X_N$
6. $\omega^1_s \in X^1_B$, $\omega^1_t \in Y_N$
7. $\omega^1_s \in X^2_B$, $\omega^1_t \in Y_N$
8. $\omega^1_s \in Y_B$, $\omega^1_t \in Y_N$
9. $\omega^1_s \in Z^1_B$, $\omega^1_t \in Y_N$
10. $\omega^1_s \in Z^2_B$, $\omega^1_t \in Y_N$
11. $\omega^1_s \in X^1_B$, $\omega^1_t \in Z_N$
12. $\omega^1_s \in X^2_B$, $\omega^1_t \in Z_N$
13. $\omega^1_s \in Y_B$, $\omega^1_t \in Z_N$
14. $\omega^1_s \in Z^1_B$, $\omega^1_t \in Z_N$
15. $\omega^1_s \in Z^2_B$, $\omega^1_t \in Z_N$. 

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Also we can formally consider fifteen cases as the combination of \( \omega_s^2 \) and \( \omega_t^2 \) for the second pivot calculation for each case of the first pivot calculation. But as soon seen, those combination mentioned above contain the cases which need not consider. The cases 1, 6, 9 and 14 never occur. Let proof these facts. Let denote the vector of variable \( \xi_{d,k} \) which belongs to the set \( x_B^1 \) as \( x_B^1 \). Also we define the vectors \( x_B^2 \), \( y_B^1 \), \( z_B^1 \) and \( z_B^2 \) as the same manner as \( x_B^1 \). Let define the vector \( w_B \) as follows:

\[
(11) \quad w_B = m + n_0
\]

\[
\begin{pmatrix}
    x_B^2 \\
    x_B^1 \\
    y_B^1 \\
    z_B^1 \\
    z_B^2
\end{pmatrix}
\]

\[
\begin{pmatrix}
    m_1 \\
    m_2 \\
    m \cdot m_1 \\
    m_1 \\
    n_0 - m_1 - m_2
\end{pmatrix}
\]

Then the coefficient matrix \( \Lambda \) for the vector \( w_B \) can be represented as follows:

\[
\Lambda = m + n_0
\]

\[
\begin{pmatrix}
    P & Q & 0 & 0 & 0 & m_1 \\
    0 & E & 0 & 0 & 0 & m_2 \\
    R & S & -E & 0 & 0 & m-m_1, \\
    E & 0 & 0 & E & 0 & m_1 \\
    0 & 0 & 0 & 0 & E & n_0 - m_1 - m_2
\end{pmatrix}
\]

where \( E \) and \( 0 \) represent the unit matrix and the zero matrix, respectively.

Assuming that the squar matrix \( P \) is regular, we have the inverse matrix \( \Lambda^{-1} \) of \( \Lambda \) as follows:
From (12), we know that there exist no nonzero elements for \( \eta_i \in Y_N \) corresponding to \( \xi_{jk} \in Y_B \). Also, there exist no nonzero elements for \( \eta_i \in Y_N \) corresponding to \( \xi_{jk} \in Z_B^1 \) and for \( \zeta_{jk} \in Z_N \) corresponding to \( \xi_{jk} \in Z_B^1 \). Hence, we cannot replace \( \xi_{jk} \in Y_B \) by \( \eta_i \in Y_N \) (the case 6), \( \xi_{jk} \in Z_B^1 \) by \( \eta_i \in Y_N \) (the case 9) and \( \xi_{jk} \in Z_B^1 \) by \( \zeta_{jk} \in Z_N \) (the case 14). For the purpose of proving that the case 1 never occurs, let denote the column vector of coefficients of the variable \( \xi_{jk} \in X_N \) as \( P_{jk} \). Then \( P_{jk} \) is represented as follows:

\[
P_{jk} = \begin{pmatrix}
p \\
m_1 \\
o \\
m_2 \\
q \\
m_1 \\
r \\
m_0 - m_1 - m_2
\end{pmatrix}
\]

As the column vector \( P_{jk}^* \) of the current simplex tableau becomes \( \Lambda^{-1} P_{jk} \), \( P_{jk}^* \) is represented as follows:
From (13), we know that there exist no nonzero elements for $\xi_{jk} \in X_N$ corresponding to $\xi_{jk} \in X_B$. Hence we cannot replace $\xi_{jk} \in X_B$ by $\xi_{jk} \in X_N$. Thus we know that the cases 1, 6, 9 and 14 never occur.

And yet, for the case 7, 11 and 12, we know that it is enough to investigate the value $\Delta H_1$. Therefore we may consider the combinations of remaining eight cases for the twice pivot calculations. But if the basic variable which moves to the nonbasis in the second iteration does not concern the fixed charge, it is meaningless for the purpose of decrease in the objective function. After the consideration of these facts, meaningful twice pivot calculations in our algorithm become as Table 1.

Table 1. Meaningful pivot calculations

<table>
<thead>
<tr>
<th>the first pivot calculation</th>
<th>the second pivot calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $\omega_s \in X_B^2$, $\omega_t \in X_N$</td>
<td>1. $\omega_s \in X_B^2$, $\omega_t \in X_N$</td>
</tr>
<tr>
<td>2. $\omega_s \in Y_B^2$, $\omega_t \in X_N$</td>
<td>2. $\omega_s \in X_B^2$, $\omega_t \in Y_N$</td>
</tr>
<tr>
<td>3. $\omega_s \in Z_B^1$, $\omega_t \in X_N$</td>
<td>3. $\omega_s \in X_B^1$, $\omega_t \in Z_N$</td>
</tr>
<tr>
<td>4. $\omega_s \in Z_B^2$, $\omega_t \in X_N$</td>
<td>4. $\omega_s \in X_B^2$, $\omega_t \in Z_N$</td>
</tr>
<tr>
<td>5. $\omega_s \in Y_B^2$, $\omega_t \in Y_N$</td>
<td></td>
</tr>
<tr>
<td>6. $\omega_s \in Z_B^2$, $\omega_t \in Y_N$</td>
<td></td>
</tr>
<tr>
<td>7. $\omega_s \in Y_B^2$, $\omega_t \in Z_N$</td>
<td></td>
</tr>
<tr>
<td>8. $\omega_s \in Z_B^1$, $\omega_t \in Z_N$</td>
<td></td>
</tr>
</tbody>
</table>
In Table 1, when we choose the variable \( w_t \in X \) to enter the basis for the first or the second pivot calculation, it is apparent that we should only consider \( \xi_{j+k} \) as the candidate variable of \( w_t \) for each \( k (k=1,\ldots,L) \), where we assume that \( \xi_{j+k} \in (X_1^1 \cup X_2^2) (k=1,\ldots,L) \). Also, when we choose the variable \( w_t \in Z \) to enter the basis, we should only consider \( \xi_{j+k} \) as the candidate variable of \( w_t \) for each \( k (k=1,\ldots,L) \), where we assume that \( \xi_{j+k} \in (Z_1^1 \cup Z_2^2) (k=1,\ldots,L) \). Moreover, if a basic variable \( \xi_{j+k} \) is chosen to be replaced by \( \xi_{h+k} \), it is apparent that the objective function does not decrease. So we may delete such cases in the pivot calculations. Also we may delete to replace a basic variable \( \xi_{j+k} \) by \( \xi_{h+k} \).

4. Algorithm

In this section, we will propose the computational procedure to solve the fixed charge problem defined as Problem B. The fundamental idea is based on the simplex method of linear programming. Though this algorithm seems to resemble the heuristic method proposed by Steinberg [6] and Walker [7], it is slightly different from [6] and [7] in respect of selecting the pivot element by utilizing properties of the problem. We prepare two kinds of simplex tableau for the algorithm, that is, Simplex Tableau 1 (ST1) and Simplex Tableau 2 (ST2). We use ST1 for the pivot calculations when variables to enter the basis and move to the nonbasis are determined. On the other hand we use ST2 only for the purpose of calculation of \( H_2 \). The basic computational procedure is constructed from Step 1 to Step 14. We use ST1 from Step 1 to Step 6 and Step 13 to Step 14, ST2 from Step 7 to Step 12 in the algorithm.

Step 1. Solve Problem A by using ordinary simplex method.
Step 2. Set \( j + 1, F \rightarrow \infty \), and \( \mu + 1 \).

Step 3. If \( \omega_j \in W_B \), go to Step 13. Otherwise determine the basic variable
\[ \omega_{i_1} \in W_B \text{ according to (14)}, \]

\[ \theta_{i_1} = \min_{1 \leq i \leq m+n_0} \left\{ \theta_i = \frac{\beta_i^1}{\sigma_{i,j}^1}, \text{ for } \sigma_{i,j}^1 > 0 \right\} \]

where \( \sigma_{i,j}^1 \) is the \((i,j)\) element of current \( ST1 \).

Step 4. Calculate the value \( \Delta H_1 \) by (15), that is,
\[ \Delta H_1 = (\gamma_j^1 - \pi_j^1) \theta_{i_1} + (\delta_j^1 - \delta_{i_1}^1). \]

Step 5. For \( \Delta H_1 \):
   a) if \( \Delta H_1 < 0 \), then go to Step 6,
   b) if \( \Delta H_1 \geq 0 \), then go to Step 7.

Step 6. Compare \( \Delta H_1 \) with \( F \):
   a) if \( F > \Delta H_1 \), set \( F = \Delta H_1 \), \( s_1 = i_1 \), \( t_1 = j \), \( \mu = 2 \) and go to Step 13,
   b) if \( F \leq \Delta H_1 \), go to Step 13 at once.

Step 7. Set each element of \( ST2 \) as the same value as \( ST1 \).

   Replace \( \omega_{i_1} \in W_B \) by \( \omega_j \in W_B \) and set \( k + 1 \).

Step 8. If \( \omega_k \in W_B \), go to Step 12. Otherwise determine the basic variable
\[ \omega_{i_2} \in W_B \text{ according to (16)}, \]

\[ \theta_{i_2} = \min_{1 \leq i \leq m+n_0} \left\{ \theta_i = \frac{\beta_i^2}{\sigma_{i,k}^2}, \text{ for } \sigma_{i,k}^2 > 0 \right\}, \]

where \( \sigma_{i,k}^2 \) is the \((i,k)\) element of current \( ST2 \).

Step 9. Calculate the value of \( \Delta H_2 \) by (17):
\[ \Delta H_2 = \Delta H_1 + (\gamma_k^2 - \pi_k^2) \theta_{i_2} + (\delta_k^2 - \delta_{i_2}^2). \]

Step 10. For \( \Delta H_2 \):
   a) if \( \Delta H_2 < 0 \), then go to Step 11,

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b) if $\Delta H_2 \geq 0$, then go to Step 12.

**Step 11.** Compare $\Delta H_2$ with $F$:

a) if $F > \Delta H_2$, set $F \leftarrow \Delta H_2$, $S_1 \leftarrow i_1$, $t_1 \leftarrow j$, $S_2 \leftarrow i_2$, $t_2 \leftarrow k$, $u \leftarrow 3$ and go to Step 12,

b) if $F \leq \Delta H_2$, go to Step 12 at once.

**Step 12.** Set $k \leftarrow k+1$:

a) if $k \leq m+2n_0$, then go to Step 8,

b) if $k > m+2n_0$, then go to Step 13.

**Step 13.** Set $j \leftarrow j+1$:

a) if $j \leq m+2n_0$, then go to Step 3,

b) if $j > m+2n_0$, then go to Step 14.

**Step 14.** For $u$:

a) if $u = 1$, the algorithm is terminated,

b) if $u = 2$, replace $\omega_{S_1}$ by $\omega_{t_1}$ by pivoting on term $\sigma^{1*}_{S_1t_1}$ and return to Step 2,

c) if $u = 3$, replace $\omega_{S_1}$ by $\omega_{t_1}$ by pivoting on term $\sigma^{1*}_{S_1t_1}$ for the first pivot calculation and then replace $\omega_{S_2}$ by $\omega_{t_2}$ by pivoting on term $\sigma^{2*}_{S_2t_2}$ for the second pivot calculation. Return to Step 2.

If various methods are contrived under the consideration of the properties mentioned in chapter 3, we can improve the efficiency of the algorithm. Let $j^*_k(X)$ ($k=1,\ldots,l$) be the maximum number of subscript of $\xi_{j^*_k}$ such that $\xi_{j^*_k} \in (X^1_B \cup X^2_B)$ for each $k$ and define $J^*(X) = \{j^*_1(X),\ldots,j^*_l(X)\}$. Also let $j^*_k(Z)$ ($k=1,\ldots,l$) be the minimum number of subscript of $\zeta_{j^*_k}$ such that $\zeta_{j^*_k} \in (Z^1_B \cup Z^2_B)$ for each $k$ and define $J^*(Z) = \{j^*_1(Z),\ldots,j^*_l(Z)\}$. Store the current $J^*(X)$ and $J^*(Z)$ after solving Problem A and at any step of the pivot calculation, that is, at Step 1 and Step 14. Then we can contrive the algorithm as follows:

a) It is enough to investigate only $\xi_{j^*_k+1k} \in X_N$ for each $k$ and only $\zeta_{j^*_k-1k}$
\( \varepsilon Z_N \) for each \( k \) at Step 3 and Step 8.

b) When determining the basic variable which moves to the nonbasis by using eq. (14) in Step 3, we only consider such subscript that \( i \in J^*(X) \) for \( \omega_i \in (X^1_B \cup X^2_B) \) and \( i \in J^*(Z) \) for \( \omega_i \in (Z^1_B \cup Z^2_B) \). Also when determining the basic variable which moves to the nonbasis by using eq. (16) in Step 8, it is enough to investigate such variable that \( \omega_i \in (X^1_B \cup X^2_B) \) and \( i \in J^*(X) \).

5. Numerical Experiments

5.1 Numerical example

To illustrate the algorithm mentioned in chapter 4, we show a simple numerical example. Let consider the problem of minimizing

\[
F = \begin{bmatrix}
\frac{1}{5} & \frac{3}{5} & \frac{5}{5} & \frac{7}{5}
\end{bmatrix}
\begin{bmatrix}
\xi_{11} \\
\xi_{21} \\
\xi_{31} \\
\xi_{41}
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{6} & \frac{3}{6} & \frac{5}{6}
\end{bmatrix}
\begin{bmatrix}
\xi_{12} \\
\xi_{22} \\
\xi_{32}
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{3} & \frac{3}{3} & \frac{5}{3}
\end{bmatrix}
\begin{bmatrix}
\xi_{13} \\
\xi_{23} \\
\xi_{33}
\end{bmatrix}
+ \begin{bmatrix}
10 & 15 & 20 & 25
\end{bmatrix}
\begin{bmatrix}
\xi_{11} \\
\xi_{21} \\
\xi_{31} \\
\xi_{41}
\end{bmatrix}
+ \begin{bmatrix}
5 & 10 & 15
\end{bmatrix}
\begin{bmatrix}
\xi_{12} \\
\xi_{22} \\
\xi_{32}
\end{bmatrix}
+ \begin{bmatrix}
10 & 10 & 10
\end{bmatrix}
\begin{bmatrix}
\xi_{13} \\
\xi_{23} \\
\xi_{33}
\end{bmatrix}
\]

subject to

\[
\begin{bmatrix}
2 & 2 & 2 & 2 \\
3 & 3 & 3 & 3 \\
4 & 4 & 4 & 4
\end{bmatrix}
\begin{bmatrix}
\xi_{11} \\
\xi_{21} \\
\xi_{31} \\
\xi_{41}
\end{bmatrix}
+ \begin{bmatrix}
3 & 3 & 3 \\
2 & 2 & 2 \\
1 & 1 & 1
\end{bmatrix}
\begin{bmatrix}
\xi_{12} \\
\xi_{22} \\
\xi_{32}
\end{bmatrix}
+ \begin{bmatrix}
1 & 1 & 1 \\
5 & 5 & 5 \\
3 & 3 & 3
\end{bmatrix}
\begin{bmatrix}
\xi_{13} \\
\xi_{23} \\
\xi_{33}
\end{bmatrix} \leq \begin{bmatrix}
10 \\
18 \\
16
\end{bmatrix}
\]

\( 0 \leq \xi_{jk} \leq 1 \) (\( j=1, \ldots, n_k \); \( k=1, \ldots, 3 \)) \begin{cases} \text{if } \xi_{jk} > 0, \text{ then } \xi_{jk} = 1 \text{ for } i = 1, \ldots, j-1 (k=1, \ldots, 3), \\ \text{if } \xi_{jk} = 0, \text{ then } \xi_{jk} = 0 \text{ for } i = j+1, \ldots, n_k \end{cases}

where \( n_1 = 4, n_2 = n_3 = 3 \) and \( \xi_{jk} = \begin{cases} 1 \text{ if } \xi_{jk} > 0 \\ 0 \text{ if } \xi_{jk} = 0 \end{cases} \) (\( j=1, \ldots, n_k \); \( k=1, \ldots, 3 \)).

The optimal state of Problem A is shown in Table 2 and Figure 3.
Table 2. Optimal tableau of Problem $A$

<table>
<thead>
<tr>
<th>$d_B$</th>
<th>$a_B$</th>
<th>$\omega_B$</th>
<th>$b$</th>
<th>$\xi_{11}$</th>
<th>$\xi_{21}$</th>
<th>$\xi_{31}$</th>
<th>$\xi_{12}$</th>
<th>$\xi_{22}$</th>
<th>$\xi_{32}$</th>
<th>$\xi_{13}$</th>
<th>$\xi_{23}$</th>
<th>$\xi_{33}$</th>
</tr>
</thead>
<tbody>
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<td>$\frac{1}{6}$</td>
<td>$\xi_{12}$</td>
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</tr>
<tr>
<td>10</td>
<td>$\frac{3}{5}$</td>
<td>$\xi_{22}$</td>
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<td>-11/30</td>
<td>-2/3</td>
<td>7/30</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>10</td>
<td>$\frac{1}{3}$</td>
<td>$\xi_{13}$</td>
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<td>1</td>
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</tr>
<tr>
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<td>$\xi_{11}$</td>
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<td>$\xi_{21}$</td>
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<td>$\xi_{31}$</td>
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<td>-13/30</td>
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<td></td>
<td></td>
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<td>$\xi_{23}$</td>
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<td>-1/3</td>
<td>1/6</td>
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<td></td>
<td>-1</td>
<td></td>
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<td></td>
<td></td>
<td>$\zeta_{22}$</td>
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<td>-1</td>
<td>11/30</td>
<td>1/15</td>
<td>-7/30</td>
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<td>-4/15</td>
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<td></td>
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<td>$\pi_{jk}$</td>
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<td>2/3</td>
<td>1/20</td>
<td>1/10</td>
<td>3/20</td>
<td>1/20</td>
<td>3/5</td>
<td>2/5</td>
<td>1/3</td>
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<td>$\Delta H_1$</td>
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<td>76</td>
<td>227</td>
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<td>32</td>
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<td>13</td>
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</tbody>
</table>
Fig. 3 Optimal state of Problem A
The value of the objective function is $F = 82.5$. According to the algorithm, we have the values of $\Delta H_1$ and $\Delta H_2$ as Table 3.

### Table 3. Values of $\Delta H_1$ and $\Delta H_2$ for Table 2

<table>
<thead>
<tr>
<th>the first pivot calculation</th>
<th>the second pivot calculation</th>
<th>values of $\Delta H_1$ and $\Delta H_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1$</td>
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<td>$\Delta H_1$</td>
</tr>
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<td>$\omega_4$</td>
<td>$\Delta H_2$</td>
</tr>
<tr>
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<td>$\eta_1$</td>
<td>$\zeta_{32}$</td>
</tr>
<tr>
<td>$\zeta_{23}$</td>
<td>$\eta_2$</td>
<td>$\zeta_{31}$</td>
</tr>
<tr>
<td>$\zeta_{22}$</td>
<td>$\eta_3$</td>
<td>$\zeta_{31}$</td>
</tr>
</tbody>
</table>

* Minimum value of $\Delta H_1$ and $\Delta H_2$.

From Table 3, we know that we should replace $\zeta_{22} \in Z_B^2$ by $\eta_2 \in X_N$ for the first pivot calculation and then $\zeta_{31} \in X_B^2$ by $\eta_1 \in X_N$ for the second pivot calculation. After these pivot calculations we have the state shown in Table 4 and Figure 4.

The value of the objective function is $F = 62.8$. For Table 4, we have the values of $\Delta H_1$ and $\Delta H_2$ as Table 5. As there exists no $\Delta H_1$ or $\Delta H_2$ which takes the negative value, we know that Table 4 shows the terminal state of Problem B.
Table 4. Terminal tableau of Problem $B$

<table>
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<th>20</th>
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<th>5</th>
<th>10</th>
<th>15</th>
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<th>10</th>
<th>10</th>
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<th>10</th>
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</thead>
<tbody>
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<td>3/5</td>
<td>5/5</td>
<td>?/5</td>
<td>1/3</td>
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<td>1/3</td>
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<tr>
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<td>-4</td>
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<td>4</td>
<td>4</td>
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<td>1</td>
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<td>3</td>
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<tr>
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<tr>
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<td>22</td>
<td>622</td>
<td>16</td>
<td>241</td>
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<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

* We assume that each empty element in the tableau takes the value zero.

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Fig. 4 Terminal state of Problem B

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Table 5. Values of $\Delta H_1$ and $\Delta H_2$ for Table 4

<table>
<thead>
<tr>
<th>the first pivot calculation</th>
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<th>values of $\Delta H_1$ and $\Delta H_2$</th>
</tr>
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</tbody>
</table>

5.2 Results of numerical experiments

In order to test the effectiveness of our algorithm mentioned in this paper, we prepare some numerical examples with the following properties:

(1) $l = 5$, $m = 5$, $n_k = 5$ ($k=1,\ldots,l$), hence $n_0 = 25$, $m + n_0 = 30$ (the number of inequalities) and $m + 2n_0 = 55$ (the number of variables including slack),

(2) $\alpha_{tj-1k} = \alpha_{tjk}$ ($i=1,\ldots,m$; $j=2,\ldots,n_k$; $k=1,\ldots,l$) and $0 \leq \alpha_{t1k} \leq 9$ ($i=1,\ldots,m$; $k=1,\ldots,l$),

(3) $\beta_{tjk}$ are given about 0.6 times as many as $\sum_j \alpha_{tjk}$ for all $i$ and $k$ ($i=1,\ldots,m$; $k=1,\ldots,l$), and

(4) $\delta_{tjk} \geq \gamma_{tjk}$ ($j=1,\ldots,n_k$; $k=1,\ldots,l$).

Table 6 shows the input data we used. For each coefficients matrix and vector ($A_k, b$) of Table 6 (a), we examined all the cases of the cost and the fixed charge vector ($c_k, d_k$) of Table 6 (b), that is, we solved $5 \times 4 = 20$ cases. Table 7 shows the results of numerical experiments. In Table 7, case 1-2, for example, implies that data no. 1 of Table 6 (a) and data no. 2 of Table 6 (b) are combined.
Table 6. Input data for the numerical experiments

(a) Coefficients matrix and vector \((A_k, b)\)

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<th>(A_1)</th>
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(b) Cost and fixed charge vector \((c_k, d_k')\)

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Table 7. Results of numerical experiments *

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* The computer used is the HITAC 8700 with OS/7 at the Computer Center of Hiroshima University except for the case 5.

‡ The value of the objective function eq. (1) for the solution of Problem A.

† Total number of pivot calculations (the number of once pivot calculation (the case μ = 2), the number of twice pivot calculations (the case μ = 3)).

§ The computer used for the case 5 is the HITAC M-180 with VOS3.
6. Conclusions

We propose an approximate solution method for the problem defined in the introduction. As the algorithm mentioned in chapter 4 is based on the simplex procedure, we can easily treat our problem. If we are in the situation in which the more precise solution must be determined, we will prepare three kinds of simplex tableau for the algorithm and define the decrease in the objective function after three times of the pivot calculations. But it is apparent that the more the pivot calculations increase, the more the computational time and the computer memory required increase.

Acknowledgements

The author wishes to thank Professor Kenichi Aoki of Hiroshima University for his helpful comments, and to the referees for their kind and useful comments. This work was supported by the Zaidan Hojin Keiei Kagaku Shinko Zaidan.

References


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