

ABSTRACT**CONSTRUCTION OF THE F-, P- AND K-TREES OF
A KNAPSAK PROBLEM AND
THEIR COMPUTATIONAL EXPERIMENTS**

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Theoretically and practically fast algorithms are presented for solving the feasibility and periodicity problems of an equality constraint knapsack problem and solving it for all right hand side numbers. Unlike usual inequality (\leq) constraint knapsack problems, equality constraint knapsack problems are not trivial even in determining their feasibility. The F-tree is a tree containing a necessary and sufficient information for the feasibility of a given knapsack problem, which is defined as a shortest path spanning tree of a certain graph. The algorithm for constructing an F-tree has a feature that the complexity of its main part does not directly depend on the number of variables involved. On the other hand, it is easy to show that the periodicity property is also held by equality constraint knapsack problems. The P-tree is a tree containing a necessary and sufficient information for the periodicity of an equality constraint knapsack problem, which is defined and obtained in the same way as the F-tree. Using the information from F- and P-trees, a method for solving the knapsack problem, whose complexity does not directly depend on the number of variables involved, can be suggested. At the stage that F- and P-trees are obtained it is found that there are only finite number of unsolved right hand side numbers which are called hard right hand side numbers in this paper and further it is possible to determine which variables are inessential to solve the knapsack problem. Two reduction methods for the inessential variables (= variables whose values are zero in an optimal solution) are associated with the F- and P-trees and they are presented. In addition to the two reduction methods, a computation saving method derived from a simple nature of numbers is employed in applying a simple dynamic programming method to solve the knapsack problems for the hard right hand side numbers. A tree structure (K-tree) is constructed over a subset of the hard right hand side numbers and the two sets of numbers associated with the F- and P-trees. The K-tree is a representation of all optimal solutions when the knapsack problem is solved for all right hand side numbers.