OPTIMAL NEW BRAND MONITORING STRATEGY FOR RETAILING

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Abstract    It is very important for supermarkets and convenience stores to detect fast and slow moving brands from a huge variety of merchandise as soon as possible. The present study proposes a new brand monitoring strategy for retailing where the sale of a new brand is monitored for a specific term \( T > 0 \) and then a judgment is made in reference to whether the brand is a fast moving brand or not. More precisely, if the number of products sold on \((0, T]\) is greater than or equal to an integer \( k \), the brand is regarded as a fast moving brand to continue its usual sale after the monitoring period. Otherwise it is regarded as a slow moving brand and its bargain sale is started to make space for another new brand or other standard brands. This study formulates the total loss incurred by the misjudgments under the above strategy, that is, a misjudgment of regarding a fast moving brand as a slow one and a misjudgment of regarding a slow moving brand as a fast one. The existence of an optimal strategy minimizing the total loss is then shown. Numerical examples are also presented to illustrate the proposed strategy.

1. Introduction
Supermarkets and convenience stores are very sensitive to new brands. This is because new brands have a possibility of being fast moving brands which make a large profit to supermarkets and convenience stores. At the same time, however, the new brands have a possibility of being slow moving brands which occupy the store space and make less profit than other standard products. For this reason, when supermarkets and convenience stores start to deal in new brands, they monitor the sale of the new brands for a relatively long term and then make a judgment in reference to whether the new brand is a fast moving brand or not.

Many studies have been reported on mathematical models associated with new brands or new products[1-5]. They are represented by new-product diffusion models[2-5] such as Bass model[3], and they were mainly conducted from the manufacturers point of view for the purpose of forecasting the demand growth. No theoretical models can, however, be seen which attempt to distinguish fast moving brands from slow moving brands from the retailing point of view.

This study proposes a new brand monitoring strategy for retailing where the sale of a new brand is monitored for a relatively short period, and then a judgment is made in reference to whether the new brand is a fast moving brand or not. In terms of brand, this study means a specific kind of products in the same category such as canned coffee. Since brands in the same category have almost same sizes, this study does not take product sizes into consideration in the following.

Depending on the demand amount of a new brand per unit time, brands are classified into three types: (1) slow moving brands, (2) fast moving brands and (3) standard brands. Definitions of these types of brand are first presented along with that of a bargain sale.
Based on these definitions, we consider a new brand monitoring strategy which starts to deal in \( m \) products of a new brand to monitor their sale for a prespecified period \( T(>0) \). When the number of products sold on \((0,T]\) are greater than or equal to a non-negative integer, \( k \), we regard the brand to be a fast moving brand, and continue its usual sale after the monitoring period. On the contrary, when the number of products sold on \((0,T]\) is less than \( k \), the new brand is regarded as a slow moving brand. Once the new brand is regarded as a slow moving brand, a bargain sale with its price reduced is started to sell out the new brand with a view to making store space for another new brand or other standard brands.

Under the proposed strategy, there is a possibility of two kinds of misjudgments, that is, (i) a misjudgment of regarding a fast moving brand as a slow moving one and (ii) a misjudgment of regarding a slow moving brand as a fast moving one. The total loss incurred by these misjudgments is then formulated to be minimized. The existence of an optimal integer \( k^* \) is also shown. Numerical examples are presented to illustrate the proposed strategy.

2. Assumptions
In this study, we assume:

(1) The accumulated number of products of a new brand demanded in a usual sale follows a Poisson process \([3,4]\) with a parameter \( \lambda \).

(2) In a bargain sale, the accumulated number of products of the new brand demanded follows a Poisson process with a parameter \( \delta \).

The parameters \( \lambda \) and \( \delta \) in the above signify demand rates of products in a usual sale and in a bargain sale respectively, where a demand rate expresses a mean number of products demanded per unit time.

From the above assumptions, we have

\[
p_i(\lambda t) \equiv \Pr[N_1(t) = i] = \frac{\lambda^i t^i}{i!} e^{-\lambda t}, \quad i = 0, 1, 2, \ldots, \tag{2.1}
\]

\[
p_i(\delta t) \equiv \Pr[N_2(t) = i] = \frac{\delta^i t^i}{i!} e^{-\delta t}, \quad i = 0, 1, 2, \ldots, \tag{2.2}
\]

where \( N_1(t) \) and \( N_2(t) \) are random variables denoting the number of products of the new brand demanded on \((0,t]\) in a usual sale and in a bargain sale, respectively.

3. Definitions
This section presents definitions of three types of brand. A concept of a bargain sale is also defined.

3.1. Three types of brand
Let \( \alpha_1 \) denote the gross profit per product in a usual sale. Let \( \beta \) express a cost for a brand to occupy the store space per unit time. Using these notations, definitions of a fast and a slow moving brands are given below:

**Definition 1** If the brand in a usual sale has a demand rate \( \lambda \) satisfying

\[
\alpha_1 - \frac{\beta}{\lambda} > 0,
\]

it is called a fast moving brand.
On the contrary, if the brand in a usual sale has a demand rate \( \lambda \) which satisfies

\[
\alpha_1 - \frac{\beta}{\lambda} < 0,
\]

it is called a slow moving brand. Finally, if the brand in a usual sale has a demand rate satisfying

\[
\alpha_1 - \frac{\beta}{\lambda} = 0,
\]

it is called a standard brand.

Let \( A = A_0 \) satisfy Eq. (3.3), and then the cost \( \beta \) for occupying the store space per unit time is given by

\[
\beta = \alpha_1 A_0. \tag{3.4}
\]

In the following, let \( \Lambda_1 \) and \( \Lambda_2 \) respectively denote sets consisting of demand rates which satisfy Inequalities (3.1) and (3.2), i.e.,

\[
\Lambda_1 = \{ \lambda | \alpha_1 - \frac{\beta}{\lambda} > 0 \}, \tag{3.5}
\]

\[
\Lambda_2 = \{ \lambda | \alpha_1 - \frac{\beta}{\lambda} < 0 \}. \tag{3.6}
\]

3.2. Bargain Sale

If a newly introduced brand is a slow moving brand, it should be sold out as soon as possible with a view to making the space for another new brand or other standard brands since products of a slow moving brand waste cost for occupying the store space. For the purpose of selling out such a slow moving brand, a bargain sale with its price reduced will be effective.

**Definition 2 (Bargain Sale)** The sale of a brand with its price reduced for the purpose of selling it out is called a bargain sale. The bargain sale considered here is continued until all products of the brand are sold out.

Let \( a_2 \) denote the gross profit per product in a bargain sale, that is, the price of the brand is reduced by \( \alpha_1 - a_2 \) in a bargain sale. Then, we have the following theorem concerning a bargain sale.

**Theorem 1** Let \( \lambda_1 (\in \Lambda_1) \) and \( \lambda_2 (\in \Lambda_2) \) respectively signify the demand rates for a fast and a slow moving brands in a usual sale. Then the following inequalities hold if a bargain sale is effective to the slow moving brand and is ineffective to the fast moving brand:

\[
\alpha_1 - \frac{\beta}{\lambda_2} < \alpha_2 - \frac{\beta}{\delta_2}, \tag{3.7}
\]

\[
\alpha_2 - \frac{\beta}{\delta_1} < \alpha_1 - \frac{\beta}{\lambda_1}, \tag{3.8}
\]

where \( \delta_1 \) and \( \delta_2 \) respectively express demand rates of the fast and the slow moving brands in a bargain sale.
Let us consider a situation where \( j (j = 1, 2, \cdots, m) \) products among \( m \) of a slow moving brand remain unsold up to \( T \) and a bargain sale is started immediately after the monitoring period. In addition, let a random variable, \( Y \), signify the bargain sale period for the remaining products, then the expected bargain sale period, \( E(Y) \) is given by

\[
E[Y] = \int_0^\infty \delta_2 x p_j(\delta_2 x) \, dx = \frac{j}{\delta_2}.
\]  

(3.9)

Hence, the expected profit by selling out the remaining products of the slow moving brand is given by

\[
j\alpha_2 - \frac{j\beta}{\delta_2} = j \left( \alpha_2 - \frac{\beta}{\delta_2} \right).
\]  

(3.10)

When a usual sale is continued for \( j \) products mentioned before, the expected profit by selling all of them in a usual sale is, likewise, given by

\[
j\alpha_1 - \frac{j\beta}{\lambda_2} = j \left( \alpha_1 - \frac{\beta}{\lambda_2} \right).
\]  

(3.11)

If the bargain sale is effective for the slow moving brand, the expected profit given by Eq. (3.10) is greater than that by Eq. (3.11), and thus we have

\[
\alpha_1 - \frac{\beta}{\lambda_2} < \alpha_2 - \frac{\beta}{\delta_2}.
\]  

(3.12)

In the above, Inequality (3.7) has been proven.

Let us consider a situation, likewise, where \( j(j = 1, 2, \cdots, m) \) products of the fast moving brand are unsold up to \( T \). If a bargain sale is executed to the remaining products immediately after the monitoring period, the expected profit of the bargain sale is given by

\[
j\alpha_2 - \frac{j\beta}{\delta_1} = j \left( \alpha_2 - \frac{\beta}{\delta_1} \right).
\]  

(3.13)

When a usual sale is continued for the remainders, the expected profit by selling them out is written as

\[
j\alpha_1 - \frac{j\beta}{\lambda_1} = j \left( \alpha_1 - \frac{\beta}{\lambda_1} \right).
\]  

(3.14)

If the bargain sale is ineffective to the fast moving brand, the expected profit given by Eq. (3.13) is smaller than that by Eq. (3.14). Hence, we have

\[
\alpha_2 - \frac{\beta}{\delta_1} < \alpha_1 - \frac{\beta}{\lambda_1},
\]  

(3.15)

and Inequality (3.8) has been proven.

\[Q.E.D.\]

4. Monitoring Strategy and Misjudgments

This study considers a new brand monitoring strategy described below:

[Monitoring Strategy]

Consider to monitor the sale of \( m \) products of a new brand over period \((0, T]\). If the number of sold products in this period is greater than or equal to a non-negative integer
We regard the new brand as a fast moving brand with a demand rate \( \lambda_1 \) and continue the usual sale for the brand. Otherwise, we regard the brand as a slow moving brand with a demand rate \( \lambda_2 \) and start a bargain sale of the brand which satisfies both Inequalities (3.7) and (3.8).

Under the above monitoring strategy, there is a possibility of two kinds of misjudgments as follows:

1. A misjudgment of regarding a fast moving brand as a slow moving one. Such a misjudgment occurs when the number of sold products on \((0, T]\) becomes less than \(k\) although the actual demand rate is \( \lambda = \lambda_1 \).

2. A misjudgment of regarding a slow moving brand as a fast moving one. This type of misjudgment occurs when the number of sold products in the monitoring period becomes \(k\) or more although the actual demand rate is \( \lambda = \lambda_2 \).

In the following, misjudgments (1) and (2) in the above are called Type I and II errors, respectively. Then the probabilities that Type I and II occur are given by

\[
\Pr[J_2|\lambda = \lambda_1] = \sum_{i=0}^{k-1} p_i(\lambda_1 T), \tag{4.1}
\]

\[
\Pr[J_1|\lambda = \lambda_2] = \sum_{i=k}^{\infty} p_i(\lambda_2 T), \tag{4.2}
\]

where \( J_1, J_2 \) signifies to regard the new brand as a fast and a slow moving brands, respectively, and

\[
\sum_{i=k}^{-1} = 0. \tag{4.3}
\]

5. Total Loss

5.1. Loss by Type I error

When a new brand is actually a fast moving brand, a bargain sale should not be conducted. Under the monitoring strategy defined in the previous section, however, the new brand will be regarded as a slow moving brand with probability \( \Pr[J_2|\lambda = \lambda_1] \) given by Eq. (4.1). In this case, a bargain sale is started for the remaining products immediately after the monitoring period. This is Type I error. A usual sale will be continued after the monitoring period with probability \( 1 - \Pr[J_2|\lambda = \lambda_1] \). Hence, the expected profit by dealing in \( m \) products of the new brand is given by

\[
A_1(k, T) = \sum_{i=0}^{k-1} \left[ i\alpha_1 + (m - i)\alpha_2 - \beta \left( T + \frac{m-i}{d_1} \right) \right] p_i(\lambda_1 T)
+ \sum_{i=k}^{m-1} \left[ m\alpha_1 - \beta \left( T + \frac{m-i}{\lambda_1} \right) \right] p_i(\lambda_1 T)
+ \sum_{i=m}^{\infty} \left[ m\alpha_1 p_i(\lambda_1 T) - \frac{m\beta}{\lambda_1} p_{i+1}(\lambda_1 T) \right], \quad k = 0, 1, 2, \cdots, m, \tag{5.1}
\]

where

\[
\sum_{i=m}^{m-1} = 0. \tag{5.2}
\]
The first term of the right-hand-side of Eq. (5.1) signifies the conditional expected profit in case the number, \( i \), of products sold on \((0, T] \) is smaller than \( k \). In this case a bargain sale for the remaining \((m - i)\) products is conducted. The second term expresses the conditional expected profit when the number, \( i \), of products sold on \((0, T] \) satisfies \( k \leq i < m \). In such a case, the usual sale is continued until all the remaining products are sold out. The third term indicates the conditional expected profit when all the \( m \) products are sold out up to \( T \).

On the other hand, if we do not conduct a bargain sale regardless of the number of products sold in the monitoring period, the expected profit by dealing in \( m \) products of the new brand is given by

\[
B_1(k, T) = \sum_{i=0}^{m-1} \left[ m\alpha_1 - \beta \left( T + \frac{m-i}{\lambda_1} \right) \right] p_i(\lambda_1 T) + \sum_{i=m}^{\infty} \left[ m\alpha_1 p_i(\lambda_1 T) - \frac{m\beta}{\lambda_1} p_{i+1}(\lambda_1 T) \right], \quad k = 0, 1, 2, \ldots, m. \tag{5.3}
\]

The first term of the right-hand-side of Eq. (5.3) expresses the conditional expected profit when the number, \( i \), of sold products in the monitoring period is smaller than \( m \). The second term signifies the conditional expected profit in case all the \( m \) products are sold out in the monitoring period.

Since we are considering a case where the new brand is a fast moving brand in this subsection, \( B_1(k, T) \) given by Eq. (5.3) expresses the expected profit under the best judgment, which will be impossible in a real situation. Then it is trivial that \( B_1(k, T) \geq A_1(k, T) \) due to Type I error. Hence the loss incurred by Type I error can be given by

\[
C_1(k, T) = B_1(k, T) - A_1(k, T) = \left[ \alpha_1 - \alpha_2 + \beta \left( \frac{1}{\delta_1} - \frac{1}{\lambda_1} \right) \right] \sum_{i=0}^{k-1} (m-i)p_i(\lambda T), \quad k = 0, 1, 2, \ldots, m. \tag{5.4}
\]

### 5.2. Loss by Type II error

When the new brand is actually a slow moving brand, we will regard the brand as a fast moving brand with the probability \( \Pr[J_1|\lambda = \lambda_2] \) given by Eq. (4.2). In this case, a usual sale is continued after the monitoring period. This is Type II error. A bargain sale will be started immediately after the monitoring period with probability \( 1 - \Pr[J_1|\lambda = \lambda_2] \). The expected profit in this case is given by

\[
A_2(k, T) = \sum_{i=0}^{k-1} \left[ i\alpha_1 + (m-i)\alpha_2 - \beta \left( T + \frac{m-i}{\delta_2} \right) \right] p_i(\lambda_2 T) + \sum_{i=k}^{m-1} \left[ m\alpha_1 - \beta \left( T + \frac{m-i}{\lambda_2} \right) \right] p_i(\lambda_2 T) + \sum_{i=m}^{\infty} \left[ m\alpha_1 p_i(\lambda_2 T) - \frac{m\beta}{\lambda_2} p_{i+1}(\lambda_2 T) \right], \quad k = 0, 1, 2, \ldots, m. \tag{5.5}
\]

On the other hand, if we start a bargain sale for the new brand immediately after the monitoring period regardless of the result in the monitoring period, the expected profit is

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given by

\[ B_2(k, T) = \sum_{i=0}^{m-1} \left[ i\alpha_1 + (m-i)\alpha_2 - \beta \left( T + \frac{m-i}{\delta_2} \right) \right] p_i(\lambda_2 T) \]

\[ + \sum_{i=m}^{\infty} \left[ m\alpha_1 p_i(\lambda_2 T) - \frac{m\beta}{\lambda_2} p_{i+1}(\lambda_2 T) \right], \quad k = 0, 1, 2, \ldots, m. \] (5.6)

As a slow moving brand is considered here, \( B_2(k, T) \) given by Eq. (5.6) expresses the expected profit under the best judgment, which will be impossible in a real situation. It is then trivial that \( A_2(k, T) \leq B_2(k, T) \) due to Type II error, and the loss incurred by Type II error can be given by

\[ C_2(k, T) = B_2(k, T) - A_2(k, T) \]

\[ = \left[ \alpha_2 - \alpha_1 + \beta \left( \frac{1}{\lambda_2} - \frac{1}{\delta_2} \right) \right] \sum_{i=k}^{m-1} (m-i)p_i(\lambda_2 T), \quad k = 0, 1, 2, \ldots, m. \] (5.7)

5.3. Total loss

This subsection defines the total loss under the proposed monitoring strategy as

\[ C_0(k, T) = C_1(k, T) + C_2(k, T) \]

\[ = \left[ \alpha_1 - \alpha_2 + \beta \left( \frac{1}{\delta_1} - \frac{1}{\lambda_1} \right) \right] \sum_{i=0}^{k-1} (m-i)p_i(\lambda_1 T) \]

\[ + \left[ \alpha_2 - \alpha_1 + \beta \left( \frac{1}{\lambda_2} - \frac{1}{\delta_2} \right) \right] \sum_{i=k}^{m-1} (m-i)p_i(\lambda_2 T), \] (5.8)

\[ k = 0, 1, 2, \ldots, m. \]

In the above, we have formulated the total loss under the proposed strategy. If \((k, T) = (k^*, T^*)\) minimizes the total loss, \(C_0(k, T)\), it is the optimum.

6. Optimal New Brand Monitoring Strategy

6.1. Optimization with respect to \( k \)

We here consider an optimal non-negative integer \( k^* \) minimizing \( C_0(k, T) \) for a fixed \( T \). Then we have the following theorem:

**Theorem 2** Let \( a \) be defined by

\[ a = \frac{\alpha_2 - \alpha_1 - \beta \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right)}{\alpha_1 - \alpha_2 - \beta \left( \frac{1}{\lambda_1} - \frac{1}{\delta_1} \right)} e^{(\lambda_1-\lambda_2)T}, \] (6.1)

and we have \( a > 0 \). Furthermore, the optimal strategy becomes as follows:

1. In the case of \( a \leq 1 \), we have \( k^* = 0 \). This indicates the optimal strategy is not to conduct a bargain sale for the new brand at all.
2. In the case of \( 1 < a < (\lambda_1/\lambda_2)^{m-1} \), there exists an optimal integer, \( 0 < k^* < m \), minimizing the total loss.
(3) If \((\lambda_1/\lambda_2)^{m-1} \leq a\), we have \(k^* = m\). In this case, the optimal strategy is to start a bargain sale for the new brand immediately after the monitoring period if there remain products unsold up to \(T\).

[Proof]
From Eq. (5.8), we have

\[
\Delta C_0(k, T) = C_0(k + 1, T) - C_0(k, T)
\]

\[
= \left[ \alpha_1 - \alpha_2 - \beta \left( \frac{1}{\lambda_1} - \frac{1}{\delta_1} \right) \times (m - k)p_k(\lambda_1T) \right] - \left[ \alpha_2 - \alpha_1 - \beta \left( \frac{1}{\delta_2} - \frac{1}{\lambda_2} \right) \times (m - k)p_k(\lambda_2T) \right],
\]

\[k = 0, 1, 2, \ldots, m - 1.\]

Since Theorem 1 yields

\[
\alpha_1 - \alpha_2 - \beta \left( \frac{1}{\lambda_1} - \frac{1}{\delta_1} \right) > 0,
\]

\[\Delta C_0(k, T) \geq 0\] is equivalent to

\[
\left( \frac{\lambda_1}{\lambda_2} \right)^k \geq \frac{\alpha_2 - \alpha_1 - \beta \left( \frac{1}{\delta_2} - \frac{1}{\lambda_2} \right)}{\alpha_1 - \alpha_2 - \beta \left( \frac{1}{\lambda_1} - \frac{1}{\delta_1} \right)} e^{(\alpha_1 - \lambda_2)T}. \tag{6.4}
\]

Furthermore we have \(\lambda_1 > \lambda_2\), and the left-hand-side of Inequality (6.4) is increasing with \(k\). It follows that if there exist non-negative integers for \(k\) satisfying Inequality (6.4), the minimum of such integers is an optimal solution.

From Theorem 1, we have

\[
\alpha_2 - \alpha_1 - \beta \left( \frac{1}{\delta_2} - \frac{1}{\lambda_2} \right) > 0,
\]

and then we obtain

\[a > 0.\] \tag{6.5}

On the other hand, by letting \(L(k)\) denote the left-hand-side of Inequality (6.4), we have

\[
L(0) = 1,
\]

\[
L(m - 1) = \left( \frac{\lambda_1}{\lambda_2} \right)^{m-1}. \tag{6.8}
\]

Hence, if \(L(0) = 1 \geq a\), we have \(L(k) \geq a\) and thus \(C_0(k, T)\) increases with \(k\). This reveals that the optimal integer minimizing the total loss becomes \(k^* = 0\). If \(L(0) < a < L(m - 1)\), \(C_0(k, T)\) increases and then decreases with increasing \(k\). It follows that there exists an optimal integer \(0 < k^* < m\) minimizing the total loss. In addition, if \(L(m - 1) \leq a\), we have \(L(k) \leq a\) and thus \(C_0(k, T)\) is decreasing with \(k\), where an optimal integer minimizing the total loss becomes \(k^* = m\).

[Q.E.D.]
6.2. Optimal strategy
When \( k \) is fixed to a non-negative integer, it is very difficult to explicitly show the existence of an optimal monitoring period \( T^* \) which minimizes the total loss. In case a set \( \{T_1, T_2, \cdots, T_n\} \) consisting of alternative values for the monitoring period \( T \) is given, the procedure to obtain an optimal strategy can be described as follows:

1. \( i = 1 \).
2. \( C_0(k_i, T_i) = \min_k C_0(k, T_i) \).
3. \( i = i + 1 \).
4. If \( i > n \), then go to [5]. Otherwise, go to [2].
5. Let \( C_0(k^*, T^*) = \min_{i=1,2,\ldots,n} C_0(k_i, T_i) \), then the optimal strategy is given by \((k^*, T^*)\).
6. Stop.

7. Numerical Examples

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</tbody>
</table>

Table 1 shows cases considered in this section along with \( k^* \) and \( C_0(k^*, T) \) in each case. Figure 1 shows \( C_0(k, T) \) in Cases 1-1, 1-2 and 1-3, where we have \( a \leq 1 \) and consequently \( k^* = 0 \).

Figure 2 indicates the shape of \( C_0(k, T) \) in Cases 2-1, 2-2 and 2-3 where we have \( 1 < a < (\lambda_1/\lambda_2)^{m^{-1}} \). In both Figs. 1 and 2, \( C_0(k, T) \) is actually a discrete function with respect to \( k \) but is shown continuously in \( k \) so that its shape can easily be recognized.

Figure 3 reveals the total loss in Cases 3-1, 3-2 and 3-3. In Cases 3-1 and 3-2, it holds that \( a > (\lambda_1/\lambda_2)^{m^{-1}} \) and therefore \( k^* = m = 5 \).

8. Concluding Remarks
This study proposed a new brand monitoring strategy for retailing which monitors the sale of a new brand for a relatively short period. When the number of sold products of the new brand in the monitoring period is less than a prespecified non-negative integer, \( k \), the brand is regarded as a slow moving brand to start its bargain sale. On the contrary, if the number of sold products in the monitoring period is greater than or equal to \( k \), the brand is regarded as a fast moving brand to continue its usual sale.

Definitions of a fast and a slow moving brands were given along with that of a standard brand. The bargain sale for a slow moving brand was also defined. On the basis of these
Figure 1: Total loss (Case 1)

Figure 2: Total loss (Case 2)

Figure 3: Total loss (Case 3)
definitions, the total loss incurred by misjudgments was formulated. An optimal strategy minimizing the total loss was then discussed. Numerical examples were also presented.

This paper did not consider the relationship between the price and the demand rate of the brand explicitly. The new brand monitoring strategy considering such a relationship is currently under investigation. In addition, when the prior information concerning whether a new brand will be a fast or a slow moving brand is available, a Bayesian approach is effective to use such information. This is also under investigation at present. A brand considered in this study represents a specific kind of products in the same item. It is, however, possible to extend the model so that it can deal with brands in different kinds of item. This will be in a forthcoming paper.

References


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