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A PRICE STABILIZATION MODEL IN NETWORKS

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Abstract We consider a multiagent network model consisting of nodes and edges as cities and their links to neighbors, respectively. Each network node has an agent and priced goods and the agent can buy or sell goods in the neighborhood. Though every node may not have an equal price, we show the prices will reach an equilibrium by iterating buy and sell operations. First, we present a protocol model in which each buying agent makes a bid to the lowest priced goods in the neighborhood; and each selling agent selects the highest bid, if any. Second, we derive a sufficient condition which stabilizes price in our model. We also show the equilibrium price can be derived from the total funds and the total goods for any network. This is a special case of the Fisher's quantity equation, thus we can confirm the correctness of our model. We then examine the best bidding strategy is available to our protocol. Third, we analyze stabilization time for path and cycle networks. Finally, we perform simulation experiments for estimating the stabilization time, the number of bidders and the effects of spreading funds. Our model is suitable for investigating the effects of network topologies on price stabilization.

Keywords: Economics, multiagent model, price determination, self-stabilization, auction theory

1. Introduction

Background. Conventionally, the topics of price determination have been discussed from microeconomics approach[22]. The discussion uses supply and demand, which capture the trend of economics as a whole. In the supply and demand curves, if the price is higher (resp. lower) than an equilibrium, there is excess supply (resp. excess demand) and thus the price moves to the equilibrium. At the equilibrium price, the quantity of goods sought by consumers is equal to the quantity of goods supplied by producers. No one has an incentive to alter price or quantity at the equilibrium. Thus, the price determination has been explained by a market mechanism, called a supply-demand theory.

Such an approach, however, cannot capture individual behaviors. For example, suppose that a company begins to sell some epoch-making goods. Some people want to buy them immediately even at a high price, and the other people want to buy them later at a low price. Since all the behaviors decide the total demand value, it is actually difficult to estimate the value by our intuition. So we should reconsider the approach in such a way the total individual behaviors form an overall market mechanism. A reasonable method is a multiagent approach, where each agent makes a decision without considering overall advantage. For example, a minority game and a network formation game attract attention.

The minority game[3, 4] contains non-cooperative, odd number of agents who make binary bids, whether to buy or sell, at each synchronous step. The bids are made by using the best strategy among several ones initially given. Any agent aims to choose the group of minority population because he makes a profit by sharing a great resource. The game continues infinitely and several interesting features, e.g., the cooperation of most agents are known. The model may be considered as a stock market if we incorporate a stock price in the game.

The network formation game [10, 13] contains (non-)cooperative agents who make binary decisions, whether to form or sever links. The decisions are made by considering benefits and costs of linking. Any agent aims to receive the maximum payoff supplied by connected agents. The game starts with an empty network, forming or severing links by endogenous agents, and ends with a stable state. The model is studied as a social network and what networks are stable in the sense that every agent enjoys maximal satisfaction.

Another idea, called self-stabilization, is similar to the multiagent games in the sense that each agent's behavior controls the entire system's behavior. The self-stabilization has been originally studied as recovery from transient faults in distributed systems[6]. From any initial state, self-stabilizing algorithms eventually lead to a legitimate state without any aid of external actions.

Problem. Several problems in the previous models are as follows. First, it is unclear with what price two agents trade goods. Some papers adopt the middle of the maximum buying price and the minimum selling price[21], and other papers adopt a log price[3]. However, there is no definite reason to use them. Second, it is unclear whether distinct prices reach an equilibrium. As far as we know, however, there is no multiagent model which corresponds to the supply-demand theory. Third, it is unclear at what price the distinct prices reach after trading. This is true both for the price produced by two agents and for the equilibrium price produced by overall agents. Fourth, it is unclear how different stabilization occurs depending on the network topologies. As a matter of course, no previous work copes with the network topologies. So we were motivated by this situation and driven to construct our original model.

Solution. We present a multiagent network model[15, 16], in which each agent repeatedly makes auctions and the price of goods eventually reaches an equilibrium. The network relation enables us to use an auction and a local trading. Our network model consists of nodes and edges as cities and their links to neighbors, respectively. Each node contains only one agent who represents people in the city. Agents who want to buy goods make bids to the lowest-priced node, if any, in the neighborhood. Then, agents who want to sell the goods accept the highest bid. That is, we first consider the local set of agents, and then put them together over the network.

For the issues stated in the problem above, we can give some answers by using several techniques. First, since our model incorporates the principle of an auction, it is clear to determine the trading price. We give a tentative bidding rule in our protocol, and then refer to the best bidding strategy. Second, since our model is compatible with the self-stabilization, it is clear to apply some proof techniques. We give the condition of convergence and prove its correctness. Third, since our model assumes the relation among funds, quantity and price at each node, it is clear to derive an equilibrium price after trading. We can confirm that the result coincides with the Fisher's quantity equation. Fourth, since our model contains the network structure, it is clear to investigate the effects of network topologies. We can derive the stabilization time only for path and cycle networks. Furthermore, we evaluate the stabilization time, the number of bidders and the spread of funds for several networks by simulation.

Related Work. The classical theory of price determination in microeconomics is introduced, e.g., in [22, 23]. Several economic network models have been already known [2, 10, 14]. Such models contain a bipartite structure [10, 14] or traders who play intermediary roles [2]. Agent-based stabilization has been discussed in [1, 11]. Unlike our *staying* agents, their ideas are to use mobile agents for the purpose of stabilization. The agents in our model attend sealed bid auctions. The sealed bid auction is argued in [17, 20]. It is useful in designing protocols by what price we should make a bid. Several kinds of game theoretic flavors have appeared in self-stabilization, e.g., time complexity analysis [7], relationships between Nash equilibria and stabilization [5, 12], and strategies with optimal complexity [9]. This paper appends an auction approach to such a trend. We can consider our protocol in Section 3 as a consensus algorithm. The consensus algorithm is presented in [18], and its self-stabilizing version is presented in [6, 8].

Contributions. We consider a new network model for economic agents where each can buy and sell goods in the neighborhood. Such a network model is useful for examining the price behavior in microeconomics. First, we present a protocol in which each agent always offers a fixed price without considering other bidders' strategies. Then, we consider an equilibrium price and the availability of the best bidding price. We also derive the stabilization time for special cases. Finally, we perform simulation experiments which reveal the effects of network topologies.

The rest of this paper organizes the following sections. Section 2 presents our model. Section 3 shows a sufficient condition for stabilization, and then discusses an equilibrium price and the best bidding strategy. Section 4 analyzes the stabilization time for path and cycle networks. Then, Section 5 shows some simulation results. Finally, Section 6 concludes the paper.

2. Model

Our system can be represented by a connected network $G = (V, E)$, consisting of a set of nodes V and edges E, where the nodes represent cities and a pair of neighboring nodes is linked by an edge. Let N_i be a set of neighboring nodes of $i \in V$. We assume that each node $i \in V$ has goods and their initial price may be distinct. The total amount of goods does not change. Let $p_i(t)^*$ be the price of goods at node i for the time step $t \in T = [0, 1, 2, \ldots]$. Each node $i \in V$ has exactly one staying agent a_i who buys/sells goods in the neighborhood †. Each agent a_i has funds (money) f_i and the quantity q_i of goods. The price p_i is determined by the relation between the quantity of goods and the buying power, called a *supply-demand* balance. So we simply assume two properties at each node. First, the price is proportional to the amount of funds for constant goods (Figure 1(a)), because it shows the relationship between money supply and inflation. Second, the price is inversely proportional to the amount of goods for constant funds (Figure 1(b)), because it shows the demand curve. That is,

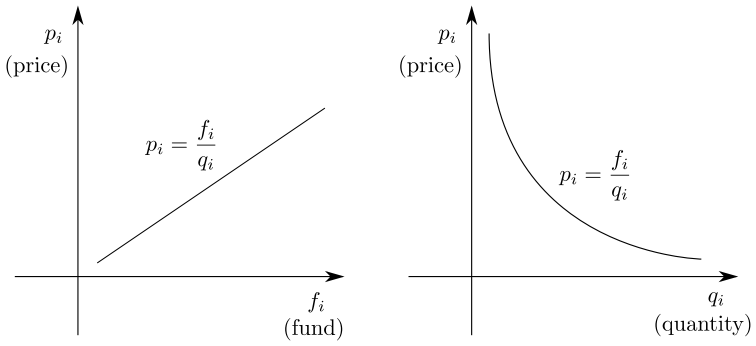
$$p_i = \frac{f_i}{q_i}. \tag{2.1}$$

The *buy operation* is executed as follows. Each agent a_i assigns a *value* $v_i^h(t)$ to the goods of any neighboring node $h \in N_i$, where the value means the maximum amount an agent is willing to pay. Agent a_i compares its own goods price p_i with any neighboring price in N_i. If the cheapest price in N_i is p_h $(< p_i)$, agent a_i wants to buy it and makes a bid b_i^h $(p_h < b_i^h)$ to node $h \in N_i$. For simplicity, we consider $v_i^h(t) = p_i(t)$ for any $h \in N_i$ because he can buy it at price $p_i(t)$ in his node[17].

The *sell operation* is executed as follows. After accepting bids from N_h, agent a_h *contracts* with a_i in N_h, an arbitrary one of agents who made the highest bid b_i^h. Then, a_h passes

*We sometimes denote the price by p_i if time t does not matter.

†We sometimes abuse agent a_i and node i like $a_i \in N_j$.



(a) Constant goods (q_i) (b) Constant funds (f_i)

Figure 1: Price determined by funds and goods at each node

his goods to (receives money from) the contracted agent a_i until the price $p_h(t+1)$ equals to $p_i(t+1)$ derived from the supply-demand balance. We do not take the carrying cost of goods into consideration but focus on the change of prices. Notice the price of goods is updated at each time step. Each node $i \in V$ has a state Σ_i represented by a tuple — the goods and the funds $(q_i(t), f_i(t))$.

We assume a *synchronous model*, that is, every agent periodically exchanges messages and knows the states of neighboring agents. We call the state of all nodes a *configuration*. We describe the set of all configurations as $\Gamma = \Sigma_1 \times \Sigma_2 \times \cdots \times \Sigma_{|V|}$. An *atomic step* consists of reading the states of neighboring agents, a buy / sell operation, and updating its own state. Then, an atomic step changes a configuration $\boldsymbol{c}_t \in \Gamma$ to $\boldsymbol{c}_{t+1} \in \Gamma$. An *execution* $\boldsymbol{E}$ is a sequence of configurations $\boldsymbol{E} = \boldsymbol{c}_0, \boldsymbol{c}_1, \ldots, \boldsymbol{c}_t, \boldsymbol{c}_{t+1}, \ldots$ such that $\boldsymbol{c}_t \in \Gamma$ changes to $\boldsymbol{c}_{t+1} \in \Gamma$.

We now consider our protocol model, called **NetBid**, where each agent a_i makes a bid b_i^h $(p_h(t) \leq b_i^h \leq p_i(t))$ to agent $a_h \in N_i$ with the lowest price in the neighborhood. Let c (≥ 1) be a constant rate, called a bid parameter, so that the bid b_i^h lies between $p_h(t)$ and $p_i(t)$. For simplicity, we assume the price or the bid may not be an integer, and we ignore a minute difference between neighboring two prices, such as $|p_i - p_j| < 1$ for $(i, j) \in E$.

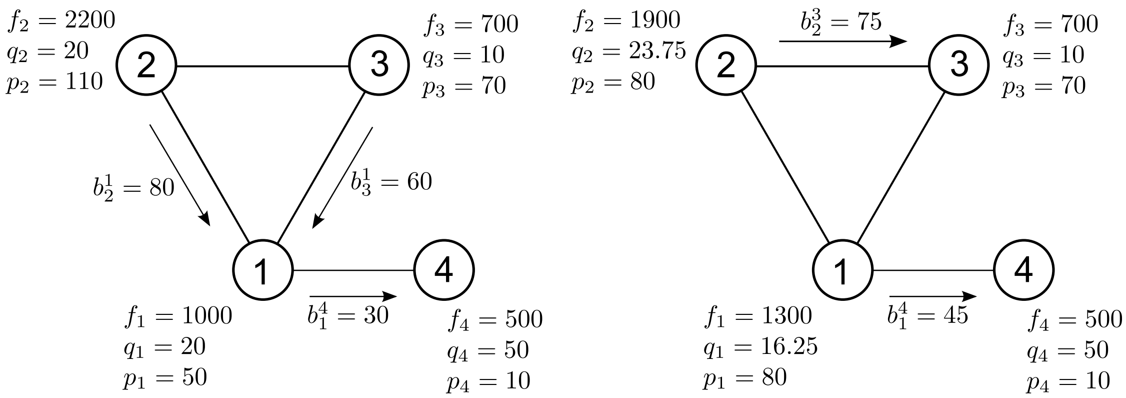
NetBid

- Each agent a_i makes a bid

$$b_i^h(t) = p_h(t) + \left(\frac{p_i(t) - p_h(t)}{c}\right) \tag{2.2}$$

  to node $h \in N_i$ which has the lowest-priced goods in N_i.
- The agent a_h contracts with the neighboring a_i who makes the highest bid $\max_{i \in N_h} b_i^h(t)$. Agent a_h's goods and a_i's funds are exchanged. More precisely, the goods moves from q_h to q_i and the funds moves from f_i to f_h as long as $p_i > p_h$. The prices $p_i(t+1)$ and $p_h(t+1)$ are determined by (2.1) at time $t+1$.
- If several agents make bids to node h with the same highest price, agent a_h makes a deal with one of them at random.
- (*priority rule:*) If concurrent buy (b_h^j to $j \in N_h$) and sell (b_i^h from $i \in N_h$) operations occur at agent a_h, he gives priority to the sell over the buy.

Example 2.1. *Figure 2 shows an example of our network system consisting of 4 nodes* $V = \{1, 2, 3, 4\}$. *Let the bid parameter* $c = 2$ *in this example. At time* t, *the prices of*



(a) Bids before trading (b) Bids after trading between 1 and 2

Figure 2: An illustration of protocol **NetBid**

goods are $(p_1(t), p_2(t), p_3(t), p_4(t)) = (50, 110, 70, 10)$ *as shown in Figure 2(a). Each agent* a_i *wants to buy the lowest-priced goods at node* $h \in N_i$ *if its price is lower than* p_i*, that is,* $p_i > \min_{h \in N_i} p_h$*. Thus, agent* a_1 *makes a bid to node 4 with price* $b_1^4 = 30$*. Similarly, agents* a_2 *and* a_3 *make bids to node 1, respectively. Since agent* a_2 *wins agent* a_3 *(i.e.,* $b_2^1 > b_3^1$*), agent* a_2 *makes a contract with agent* a_1*. Notice that* b_1^4 *is delayed by the priority rule.*

Let x *units be the number of* a_2*'s buying goods. Since the prices of nodes 1 and 2 become equal, we have*

$$\frac{1000 + 80x}{20 - x} = \frac{2200 - 80x}{20 + x}.$$

This gives $x = 3.75$ *and hence* $q_1 = 20 - x = 16.25$*,* $q_2 = 20 + x = 23.75$*,* $f_1 = 1000 + 80x = 1300$*,* $f_2 = 2200 - 80x = 1900$*, and* $p_1 = (1000 + 80 \times 3.75)/(20 - 3.75) = 80 = p_2$*.*

At time $t' = t + 1$*, the prices become* $(p_1(t'), p_2(t'), p_3(t'), p_4(t')) = (80, 80, 70, 10)$ *as shown in Figure 2(b). Since price* p_1 *has been changed, agent* a_1*'s bid* b_1^4 *is made again as* $(80 + 10)/2 = 45$*. Since the bids* b_2^3 *and* b_1^4 *are independent, they are executed in parallel at time* t'*.* □

3. Correctness and Properties

In this section, we examine our protocol model. First, we describe the correctness of our protocol in Section 3.1. Second, we find an equilibrium price based on our assumption in Section 3.2. Third, we refer to the best bidding price and its availability to our protocol in Section 3.3.

3.1. Correctness of our protocol

Our concern is whether the distinct prices of goods eventually reach an equilibrium price. So we define the legitimacy of a configuration as follows.

Definition 3.1 (legitimate configuration)**.** *A configuration is legitimate if every node has equally priced goods.* □

Let $C_t \subseteq V$ be the set of nodes that have updated their prices from time t to $t+1$. The following lemma proves the protocol **NetBid** is free from deadlocks.

Lemma 3.1. *The protocol* **NetBid** *is deadlock-free. That is, nonempty* C_t *means the configuration is illegitimate.*

Proof. First, the chain of bidding requests does not form a cycle because every bidding request occurs from a higher priced node to a lower priced node.

Next, suppose the configuration is illegitimate at time t. Then, there is a pair of neighboring nodes $i, j \in V$ with $p_i(t) = \max_{m \in N_j} p_m(t)$ and $p_j(t) = \min_{m \in N_i} p_m(t)$, where $p_i(t) - p_j(t)$ is the maximal price difference in the neighborhood. In this case, agent a_i makes a bid to node j and agent a_j accepts the price. Since $p_j(t)$ rises at time $t+1$, $j \in C_t$ holds. Since there is no cycle, the price movement occurs as long as C_t is nonempty. □

In [16], we examined a sufficient condition for price stabilization in **NetBid**. Suppose that agents a_i and a_j make bids to node $h \in N_i \cap N_j$. We say that *bids have the same order as values* if $v_i^h \leq v_j^h$ implies $b_i^h \leq b_j^h$ for the goods of node h. Next lemma shows the bids having the same order as values is necessary for price stabilization.

Lemma 3.2. *If bids do not keep the same order as values, we do not have price stabilization.*

Proof. Suppose there exists an agent a_i who has the maximum value in $h \in N_i$. That is, $v_i^h \geq v_j^h$ holds for any node $j \in N_h$. If agent a_i always makes a bid with the lowest price in the neighborhood ($b_i^h < b_j^h$), he always loses and thus the price p_i does not change. Thus, we cannot guarantee the price stabilization. □

Let $P^{max}(t) = \max_{i \in C_t} p_i(t)$ be the highest price in C_t, and $P^{min}(t) = \min_{i \in C_t} p_i(t)$ be the lowest price in C_t. Additionally, let $\mathit{diff}(t) = P^{max}(t) - P^{min}(t)$. The following theorem further shows that an additional condition leads to the price stabilization.

Theorem 3.1. *Suppose bids keep the same order as values. If any contract price lies between buyer's price and seller's price, price stabilization occurs.*

Proof. Let $v_i^h(t) = p_i(t) = P^{max}(t)$. Let node h with $\min_{h \in N_i} p_h(t)$ have the minimum price in the neighborhood. Since bids have the same order as values, agent a_i makes the highest bid in N_h. Thus agent a_i can contract with a_h. It means $P^{max}(t) = p_i(t) > p_i(t+1)$. Since no other agents make bids greater than $P^{max}(t)$, we have $P^{max}(t) > P^{max}(t+1)$. The similar argument holds for $P^{min}(t)$. Thus we have

$$\mathit{diff}(t) > \mathit{diff}(t+1). \tag{3.1}$$

This means price stabilization occurs. □

If any contract price does not lie between buyer's price and seller's price, price stabilization is not guaranteed. It is clear because the inequality (3.1) does not hold. Since we assume that $v_i^h(t) = p_i(t)$ for any neighboring node $h \in N_i$ and a_i makes a bid by (2.2), **NetBid** satisfies the condition above.

3.2. Equilibrium price

Here, we consider an equilibrium price after the stabilization based on our assumption (2.1). The following theorem claims the total amount of funds and goods settles the equilibrium price regardless of network topologies.

Theorem 3.2. *Let F be the total amount of funds, and Q the total amount of goods in a network G. The equilibrium price, denoted by P^e, is*

$$P^e = \frac{F}{Q}$$

regardless of network topologies.

Proof. By definition, the price of goods at node i is $p_i = f_i/q_i$. Suppose the equilibrium prices are different for each stabilization. That is, $P^e(t) = p_i(t) \neq p_i(t') = P^e(t')$ for time t and t' $(t \neq t')$ holds. Since $f_i = p_i(t)q_i$ and $f'_i = p_i(t')q'_i$ hold for any node i, where $F = \sum_i f_i = \sum_i f'_i$, we have

$$p_i(t) \cdot \sum_i q_i = p_i(t') \cdot \sum_i q'_i.$$

Since the total amount of goods Q is identical, we have

$$Q = \sum_i q_i = \sum_i q'_i.$$

Thus we have $p_i(t) = p_i(t')$, a contradiction. Therefore, the equilibrium price P^e is identical for each stabilization.

Next, since $f_i = P^e \cdot q_i$ holds for every node i, the total funds sum up to

$$F = P^e \cdot Q.$$

Thus we have $P^e = F/Q$. □

The theorem above is known as the *Fisher's quantity equation*[19] $FV = P^eQ$ if the velocity of funds V equals to 1. This means the correctness of our assumption (2.1) at each node.

Let $p^e(t+1)$ be the temporary, shared price of nodes i and j reached by trading exhaustively for a contract between t and $t+1$. From Theorem 3.2, we have the following corollary.

Corollary 3.1. *Let $q_i(t)$ and $q_j(t)$ be the quantity of goods, and $f_i(t)$ and $f_j(t)$ the funds before the trade, respectively. After the trade, the shared price will be*

$$p^e(t+1) = \frac{f_i(t) + f_j(t)}{q_i(t) + q_j(t)}.$$

□

3.3. Best bidding for constant bidders

In this section, we consider whether the Bayesian-Nash equilibrium [17, 23], known as the best strategy for a sealed-bid auction, is applicable to our model. Suppose that each agent's value is uniformly distributed on (α, β) (independent and identically distributed). Then, the distribution function is $F(x) = r(x - \alpha)$, where $r = 1/(\beta - \alpha)$. Let Y be the highest of $B - 1$ values. Then, Y is the highest order statistic of the values. Thus, the distribution function of Y, denoted by $G(x)$, is $G(x) = F(x)^{B-1}$. In addition, let $g(x)$ be the density function of $G(x)$. Let $B^h(t)$ be the number of bidders to node h at time t, or simply denoted by B. Agent a_i's best strategy $S(v_i^h)$ against $B - 1$ bidders is known [16, 17] as

$$\begin{aligned} S(v) &= \frac{1}{G(v)} \int_\alpha^v y g(y) dy \\ &= \frac{1}{\{r(v-\alpha)\}^{B-1}} \int_\alpha^v y r^{B-1}(B-1)(y-\alpha)^{B-2} dy \\ &= v - \frac{v - \alpha}{B} \end{aligned}$$

We now examine whether the strategy above satisfies our condition in Theorem 3.1. First, for the orders of bids and values, suppose $v_i^h \le v_j^h$ holds.

$$\begin{aligned} b_j^h - b_i^h &= S(v_j^h) - S(v_i^h) \\ &= (v_j^h - \frac{v_j^h - \alpha}{B}) - (v_i^h - \frac{v_i^h - \alpha}{B}) \\ &= (v_j^h - v_i^h)(1 - \frac{1}{B}) \ge 0. \end{aligned}$$

Thus $b_i^h \le b_j^h$ holds.

Next, for the bidding price,

$$S(v_i^h) - \alpha = v_i^h - \frac{v_i^h - \alpha}{B} - \alpha = (v_i^h - \alpha)(1 - \frac{1}{B}) > 0.$$

On the other hand, it is clear that

$$v_i^h - S(v_i^h) = v_i^h - (v_i^h - \frac{v_i^h - \alpha}{B}) = \frac{v_i^h - \alpha}{B} > 0.$$

Thus, we have $\alpha < S(v_i^h) < v_i^h$. Now we obtain the following theorem.

Theorem 3.3. *In our protocol* **NetBid***, suppose that each agent a_i confronting $B-1$ bidders repeatedly makes a bid to the lowest-price node $h \in N_i$ by strategy*

$$S(v_i^h(t)) = v_i^h(t) - \frac{v_i^h(t) - p_h(t)}{B} = p_i(t) - \frac{p_i(t) - p_h(t)}{B}.$$

Then, price stabilization occurs. □

Actually, the problem in our model is to know the precise number of bidders B. Though the maximum number of bidders to node i is N_i, some of them might place bids to other neighboring nodes. Further argument is taken over Section 5.2.

4. Some Analyses

In this section we investigate stabilization time for special networks. First, we focus on a path in Section 4.1. Next, we apply the obtained result to a cycle in Section 4.2.

4.1. Stabilization time for a path

In the sequel, we analyze the stabilization time for a path $(1, \ldots, n)$.

Example 4.1. *Figure 3 illustrates the price movement in a path. Suppose that there is a path $(1, 2, 3, 4, 5, 6)$ such that $(p_1(0), \ldots, p_6(0)) = (50, 57, 55, 60, 56, 70)$, where the prices are represented by the nodes position in the vertical direction as in Figure 3(a). The arrows indicate the price movement at the next step. After time 1 (Figure 3(b)(c)(d)), several pairs of shared prices arise, and the similar movements repeatedly occur. Note the positional relation between nodes does not change for every other time (e.g., (b) and (d)).* □

We consider only an ascending/descending sequence of prices (like Figure 3(b)(c)(d)) because it clearly takes worst case stabilization time. We assume $n = 2m$ is even. The following theorem shows the stabilization time for a path.

Theorem 4.1. *If network G is a path, the stabilization time of our* **NetBid** *is 2τ steps, where τ satisfies*

$$\left(\frac{3}{4}\right)^{\tau} \left(\frac{1}{3}\right)^{n/2+1} \binom{\tau}{n/2+1}^{n/2+1} = 1.$$

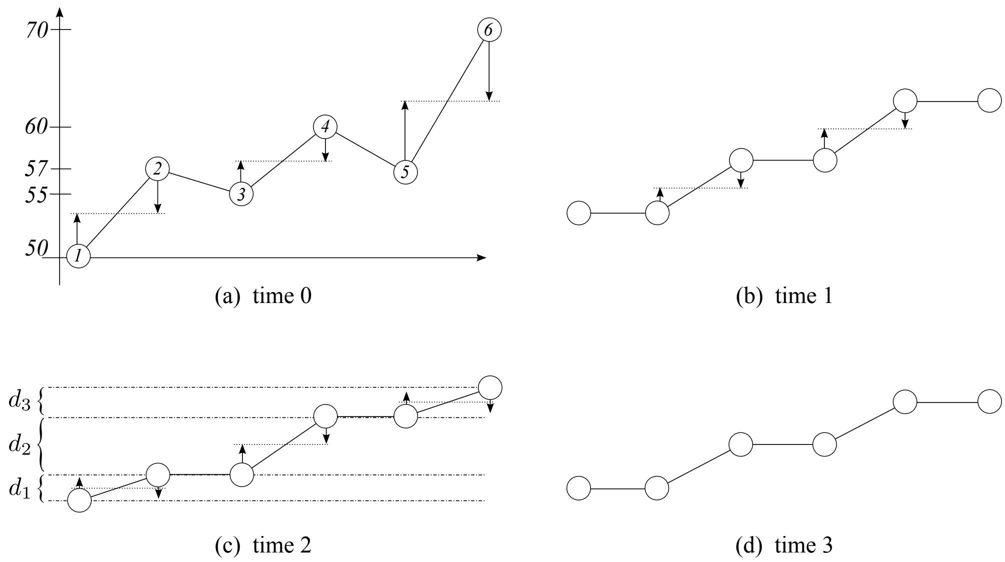


Figure 3: Price movement in a path

Proof. We call the price difference between neighboring nodes a *gap*, and call the gaps as 1st gap, 2nd gap ... in the ascending order of the nodes. Let $d_i(t)$ be the difference of the i-th gap ($1 \leq i \leq m$) at time t, where t means every other time here. Then, we have the following recurrences.

$$d_i(t+1) = \frac{1}{4}d_{i-1}(t) + \frac{1}{2}d_i(t) + \frac{1}{4}d_{i+1}(t) \tag{4.1}$$

$$d_1(t+1) = \frac{1}{2}d_1(t) + \frac{1}{4}d_2(t) \tag{4.2}$$

$$d_m(t+1) = \frac{1}{2}d_m(t) + \frac{1}{4}d_{m-1}(t) \tag{4.3}$$

Let $S_{m-j}(t) = \sum_{i=j+1}^{m-j} d_i(t)$ and $S_m(0) = D$. Summing (4.1) from $i = 1$ to m by using (4.2) and (4.3) gives

$$S_m(t+1) = S_m(t) - \frac{1}{4}(d_1(t) + d_m(t)).$$

Since $d_1(t) + d_m(t) = S_m(t) - S_{m-1}(t)$, we have

$$S_m(t+1) = \frac{3}{4}S_m(t) + \frac{1}{4}S_{m-1}(t). \tag{4.4}$$

If we define the generating function $G_t(z) = \sum_{m\geq 0} S_m(t)z^m$, Equation (4.4) can be written as

$$\sum_{m\geq 0} S_m(t+1)z^m = \sum_{m\geq 0} \frac{3}{4}S_m(t)z^m + \sum_{m\geq 1} \frac{z}{4}S_{m-1}(t)z^{m-1},$$

that is,

$$G_{t+1}(z) = \frac{3+z}{4}G_t(z) = \cdots = \left(\frac{3+z}{4}\right)^{t+1} G_0(z).$$

Since $G_0(z) = \sum_{m\geq 0} S_m(0)z^m = D\sum_{m\geq 0} z^m = \frac{D}{1-z}$, we have

$$G_t(z) = \left(\frac{3+z}{4}\right)^t \cdot \frac{D}{1-z} = \frac{D(3/4)^t(1+z/3)^t}{1-z}.$$

Hence, the coefficient of z^m is

$$
\begin{aligned}
& D(3/4)^t \sum_{k=0}^{m} (1/3)^k \binom{t}{k} \\
= \; & D(3/4)^t\{(1+1/3)^t - (1/3)^{m+1}\binom{t}{m+1} - O(1/3^{m+2})\} \\
\leq \; & D\left\{1 - \left(\frac{3}{4}\right)^t \left(\frac{1}{3}\right)^{n/2+1} \left(\frac{t}{n/2+1}\right)^{n/2+1}\right\}
\end{aligned}
$$

because $m = n/2$. Hence, $S_m(t) = 0$ gives $(\frac{3}{4})^t(\frac{1}{3})^{n/2+1}\binom{t}{n/2+1}^{n/2+1} = 1$. Since it takes $2t$ steps until convergence, the lemma follows. □

4.2. Stabilization time for a cycle

Since a path $(1, 2, \ldots, n)$ becomes a cycle if both ends, 1 and n, are connected, we can apply the result of a path. We consider an initial state where one end, 1, with the lowest price through the other end, n, with the highest price have increasing prices. Since 1 and n are connected, the lowest price and the highest price gradually shift from 1 and n to their intermediate nodes. Let us call the intermediate nodes between the lowest-price node and the highest-price node, initially $n-2$ nodes between 1 and n, a *center part*. Additionally, let us call the nodes between 1 (resp. n) and the lowest-price node (resp. the highest-price node), initially an empty set, a *left-hand part* (resp. *right-hand part*). Notice that the center part is not influenced by the price movement of the left-hand part and the right-hand part.

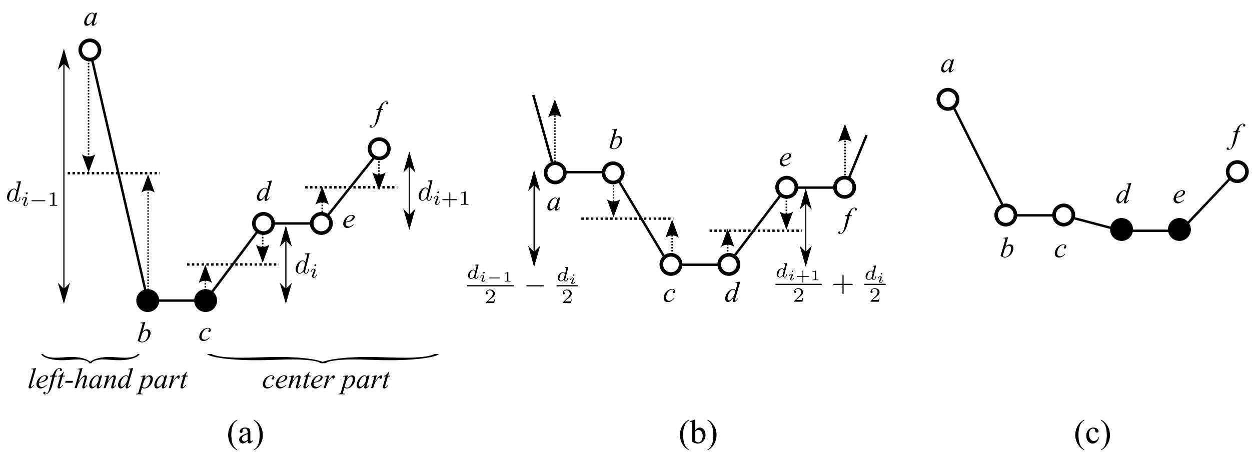


Figure 4: Shift of the lowest price

So we want to know the range of the center part whose convergence determines the stabilization time of a cycle. When does the lowest price node shift to the neighboring intermediate node ? We call the price difference between the left-neighboring (resp. right-neighboring) node and the lowest price node a *left-difference* (resp. *right-difference*), denoted by d_{i-1} (resp. d_i) in Figure 4(a). The lowest price nodes (b and c in Figure 4(a)) will be shifted to the next nodes (d and e in Figure 4(c)) if the left-difference is larger than the right-difference. The shift of the lowest price nodes continues until the left- and the right-difference become equal. Such a state will eventually occur. Thus,

$$\begin{aligned}\frac{d_{i-1}}{2}-\frac{d_i}{2} &= \frac{d_{i+1}}{2}+\frac{d_i}{2}\\ d_i &= \frac{1}{2}(d_{i-1}-d_{i+1}).\end{aligned}$$

If we assume $d_i \approx d_{i+1}$, we have $d_i = d_{i-1}/3$. This means the left-hand side difference is about 3 times as large as the center part difference. Thus, the center part length is at most $3n/4$. So we have the following corollary.

Corollary 4.1. *The stabilization time for an n-node cycle is less than that for a $\frac{3n}{4}$-node path.* □

5. Simulation

Here we present some simulation results. First, we examine the number of steps for stabilization in Section 5.1. Second, we propose two methods of estimating the number of bidders, and evaluate them for several topologies in Section 5.2. Third, we focus on the spread of funds throughout the network, and investigate them for several topologies in Section 5.3.

We first describe constants and parameters common to the following experiments. We repeat the experiment up to 500 trials, where a trial ends with an equilibrium, and obtain averaged results. We change the number of nodes from 50 to 500 with others fixed. We set the bid parameter at $c = 2$ in (2.2).

Table 1: Common constants and parameters

Meaning	Values
Number of trials	500
Number of nodes	50 — 500
Amount of funds	10,000
Amount of goods	$[50, 100]$

5.1. Stabilization time

Here we investigate how different topologies have influence on price stabilization. For the different topologies, we consider four kinds of graphs, a path, a grid, a random k-link graph, and a complete graph. The random k-link graph is defined as a cycle with randomly selected k edges. We compare the number of steps until convergence, called a stabilization time, on the graphs.

Figures 5 and 6 show the stabilization time for path vs. random k-link graphs and three networks, respectively. See Figure 5. A path and a cycle (i.e., 0-link) need much longer time than other graphs to reach an equilibrium. Since the path and the cycle contain no extra edges, they limit to spread the price. The result confirms Corollary 4.1.

See Figure 6. Since the grid graph has many 4-degree nodes, there are many trading agents concurrently. On the other hand, the complete graph has only one pair trading agents. Thus, the complete graph has longer stabilization time than the grid graph. The random $|V|/10$-link graph has an intermediate feature between the complete graph and the grid graph. Since it has randomly selected edges, the mean time is somewhat unstable.

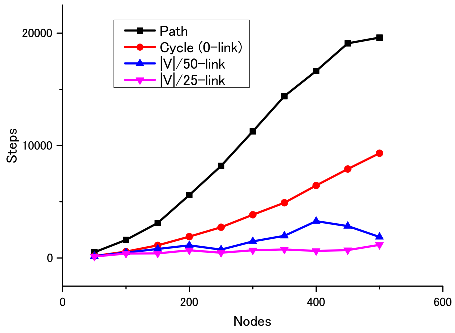


Figure 5: Stabilization time
(path vs. k-link)

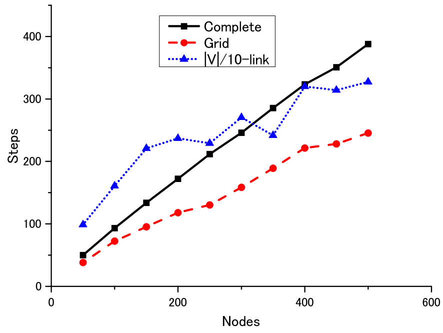


Figure 6: Stabilization time
(several networks)

5.2. Number of bidders

As shown in Section 3.3, it is necessary to know the number of bidders B for the decision of the best bidding price. Since **NetBid** does not contain such a method, we give it in the following. We compare the following two methods which enable us to estimate the number of bidders B. We examine which method is suitable for the estimation of B.

1. A method is to use B in the previous step by assuming the number of bidders is available.

2. Another method is to estimate B (by (5.1)) by assuming each agentfs value is uniformly distributed over the same interval.

Method 1 uses the previous information based on the idea that the situation would not suddenly change in time. For example, suppose agent a_i wants to estimate $B(t)$ at node $h \in N_i$. Then a_i just substitutes $B(t-1)$ for $B(t)$. Method 2 uses the neighboring information of prices based on the idea that the situation would not suddenly change in location. More precisely, let gap_h be the difference between the maximum price in $N_h \cup h$ and the minimum price in $N_h \cup h$. Let g_h be the difference between the maximum price in $N_h \cup h$ and the price of node h. Then, agent a_i estimates the number of bidders to node $h \in N_i$ as

$$e_h(B) = int\left(\frac{g_h}{\max(gap_h, 1)} \cdot |N_h|\right), \tag{5.1}$$

where "int" rounds off to an integer and $\max(gap_h, 1)$ avoids zero for a denominator.

We consider a sparse graph and a dense graph by using a random k-link graph with different k values. To evaluate the methods, we use the following expression

$$D = \sum_{i \in V} \left(\frac{e_i(B) - a_i(B)}{|N_i|}\right)^2,$$

where $a_i(B)$ is the actual number of B. We collect the value of D in each step and take an average of them. Notice that D represents a deviation from the actual number of bidders, and its range is $0 \le D \le |V|$.

Figure 7 shows the difference between the methods for two graphs. Method 2 is a little more effective than Method 1 in dense ($k = |V|/1.2$) networks. On the other hand, Method 1 is much more effective than Method 2 in sparse ($k = |V|/10$) networks. Since there are many buying choices in dense networks, the estimation like Method 2 would be useful. Conversely,

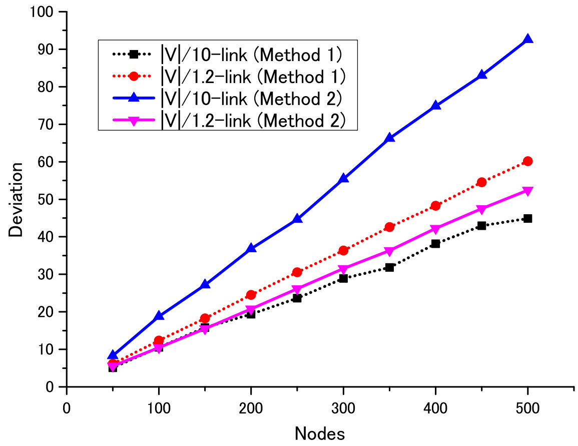


Figure 7: Estimation of number of bidders

since there are not many buying choices in sparse networks, the estimation like Method 2 would be useless.

5.3. Spread of funds

Here we consider how injected funds spread in several networks because sufficient funds of each agent drives him to buy goods. To inject funds is a monetary policy for deflation.

Our questions are :

1. How do different topologies have influence on the spread of funds ?
2. How does the number of nodes have influence on the spread of funds ?
3. How does the number of injection points have influence on the spread of funds ?, and
4. How does the number of links have influence on the spread of funds ?

The following tables summarize basic constants and some parameters. The amount of funds is 100 for a non-injection point and 30,000 for an injection point. The amount of goods is uniformly distributed between 50 and 100 for each node (Table 1). Then, the price of goods is determined by Equation (2.1).

Table 2: Basic constants

Meaning	Value
Amount of funds	100
Injection funds	30,000

Table 3: Parameters

Meaning	Value	Standard Value
Number of nodes	50—500	300
Number of injection points	1—10	1

We consider four types of networks, a path, a grid, a complete graph, a random k-link graph. The grid and the complete graph are representative sparse and dense networks, respectively. We use $k = |V|/10$ in the following experiments.

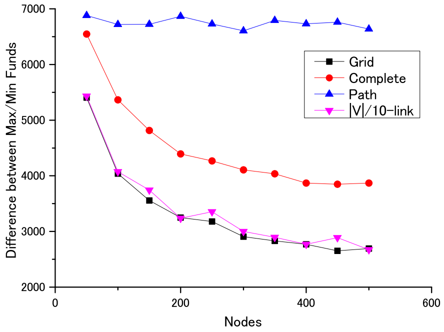


Figure 8: Max / min funds vs. number of nodes for four networks

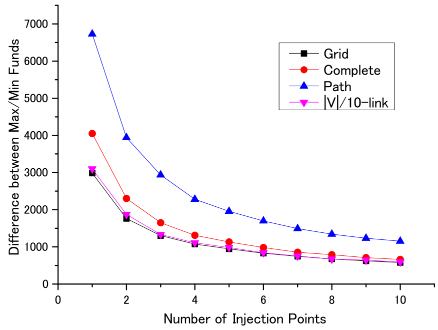


Figure 9: Max / min funds vs. number of injection points for four networks

The first experiment examines the questions No.1 and No.2, where a varying parameter is the number of nodes, changing from 50 to 500. Further, the number of injection is 1 (see Figure 8). The figure shows the grid (including the random $|V|/10$-link) is better than others from the fund-spreading view. The path is the most ineffective network because another terminal node is very distant from the injection point. Mostly, the increment of nodes is suitable for spreading funds. The complete graph and the grid hold this, while the path not. This is because the increment of nodes increases extra edges. Further, the grid has many pairs of trading nodes because they consist of local maximal/minimal price nodes. On the other hand, the complete graph has only one pair of trading nodes with maximum/minimum prices. Thus, the grid outperforms the complete graph for the spread of funds.

The next experiment examines the questions No.1 and No.3, where a varying parameter is the number of injection points, changing from 1 to 10. Further, we fix the number of nodes to 300 (see Figure 9). The injection points are node 1 and randomly selected nodes. The figure shows the grid (including the random $|V|/10$-link) is better than other networks from the fund-spreading view. The increment of injection points is effective in the spread of funds for arbitrary networks. This is clear because the fund-spreading policy would be complete if every node were equally injected funds $F/|V|$.

The last experiment examines the question No.4, where a varying parameter is the number of links k, changing from 0 to 1500. Further, we fix the number of nodes to 300 and the number of injection to 1 (see Figure 10). The figure shows that the curve sharply falls from 0 to next point and then gradually rises as the number of links increases. This is clear because the random k-link resembles the path when $k \approx 0$ and resembles the complete graph when k becomes large.

6. Conclusion

In this paper we considered a new network model for the price stabilization. First, we presented a system model, where the price is proportional to funds and the price is inversely proportional to goods at each node. Then we provided a protocol which moves funds / goods, and showed a sufficient condition that price stabilization occurs. We also showed that the equilibrium price is determined by the total amount of funds and the total amount of goods. We examined the best bidding strategy for constant bidders can be applied to our protocol. Next, we derived the stabilization time for special cases. Finally, we presented simulation

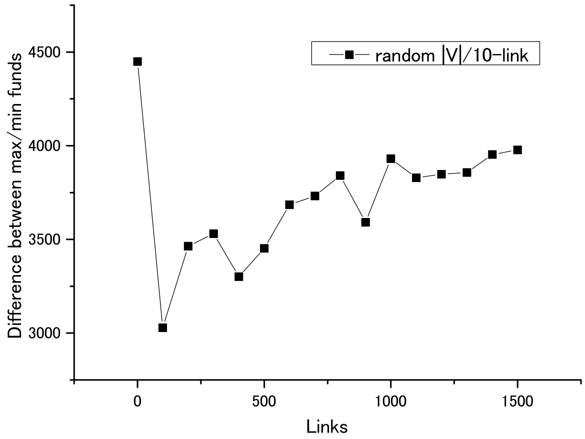


Figure 10: Max / min funds vs. number of links in random k-link

results. In summary, our network model reveals the following facts.

- The price stabilization occurs in our model if bids have the same order as values and any contract price lies between buyer's price and seller's price.
- The equilibrium price can be estimated if the price is proportional to funds (and inversely proportional to goods) at each node.
- Somewhat denser networks are easy to reach an equilibrium because there are multiple paths for spreading the prices.
- For the best bidding, the Bayesian-Nash solution needs the number of bidders B. To estimate B, our Method 2 is a little useful for a dense graph, while our Method 1 is useful for a sparse graph.
- Sparse (not extremely sparse) networks like the grid is better than dense networks like the complete graph from the fund-spreading view.

Our goal is to construct a good multiagent model which enables us to simulate a realistic social system. Then, we could analyze and estimate several economic phenomena. Our future work includes investigating an asynchronous model and developing other protocols.

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