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SPACING OF INTERSECTIONS IN HIERARCHICAL ROAD NETWORKS

Masashi Miyagawa
University of Yamanashi

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Abstract This paper presents an analytical model for examining the spacing of intersections that connect different levels of roads in a hierarchical network. An analytical expression for the average travel time is obtained for a grid road network with two road types: minor and major roads. The travel time is defined as the sum of the free travel time and the delay at intersections. The analytical expression gives basic properties of the tradeoff between the travel time on minor and major roads. The optimal pattern of intersections that minimizes the average travel time is then obtained. The result demonstrates how the road length, the intersection delay, the travel speed, and the size of the city affect the optimal pattern. The model is also applied to the road network of Tokyo. The proposed model explicitly considers the tradeoff between the accessibility to higher level roads and the delay at intersections, and is useful for hierarchical road network design.

Keywords: Transportation, network design, grid network, average travel time, accessibility, intersection delay

1. Introduction

Road networks have hierarchy ranging from low-speed access roads to high-speed arterial roads. In hierarchical road networks, intersections connecting different levels of roads significantly affect the travel time. If few intersections exist, the transfer between lower and higher level roads would be difficult. On the other hand, too many intersections increase the travel time due to the intersection delay. The optimal spacing of intersections should be considered in hierarchical road network design.

The hierarchical network design has been addressed in both discrete and continuous network models. The discrete models aim to develop efficient algorithms applicable to actual networks [4, 7, 17, 18, 20]. The discrete models usually use detailed traffic data and equilibrium traffic assignment. The continuous models, in contrast, aim to find fundamental relationships between variables. The distribution of travel demand is approximated by continuous functions. Since the continuous models yield simple closed form solutions that are easy to interpret and comprehend, they can supplement the discrete models. In the continuous models, one of the most frequently used networks as an approximation for actual networks is a grid network. The grid network model is useful for Kyoto in Japan, Beijing in China, and many cities in North America. Creighton et al. [3] considered a square grid network with three road types and determined the optimal spacing by minimizing the sum of the travel and construction costs. Tanner [21] examined the average travel time on a rectangular grid network. Fawaz and Newell [8, 9] also used a rectangular grid network and showed that an efficient network would have a hierarchical structure. Miura [13] estimated the minimum length of high-speed roads required to manage traffic flow on a grid road network. Miyagawa [14] proposed a grid network model for finding the optimal road area in a hierarchical network. The model was extended by Miyagawa [15] to incorporate inter-city traffic and Miyagawa [16] to determine the total road area.

In the continuous models concerning hierarchical network design, few studies have considered the effect of intersections. In fact, Miyagawa [14] assumed that the travel time decreases as the road length increases. This assumption is not necessarily valid because the increased intersections can cause additional delay such as signal waiting time and congestion. Thus, there exists a tradeoff between the accessibility to higher level roads and the delay at intersections. Overlooking this tradeoff would result in the over-supply of higher level roads.

In this paper, we present a continuous model for determining the optimal spacing of intersections that connect different levels of roads in a hierarchical network. To consider the tradeoff between the accessibility to higher level roads and the delay at intersections, we examine how the spacing of intersections affects the average travel time. The travel time is defined as the sum of the free travel time and the delay at intersections. The model uses a grid road network with two road types and yields an analytical expression for the average travel time. The analytical expression leads to a clear understanding of basic properties of the intersection spacing. The present model will therefore supply building blocks for hierarchical network design models.

The rest of this paper is organized as follows. The next section presents a model of a grid road network. Section 3 gives an analytical expression for the average travel time. Section 4 finds the optimal intersection pattern that minimizes the average travel time. Section 5 applies the model to the road network of Tokyo. The final section presents concluding remarks.

2. Grid Network Model

Consider a square city with side length A, as shown in Figure 1. The city has a grid road network with two road types: minor and major roads. Minor roads exist everywhere, whereas major roads are on a square grid with spacing a. Let Λ be the length of major roads, i.e., $\Lambda = 2A \cdot A/a = 2A^2/a$. The spacing a is then expressed in terms of the road length Λ as $a = 2A^2/\Lambda$. The transfer between minor and major roads are only allowed at intersections shown by white circles in Figure 1.

Assume that origins and destinations are uniformly and independently distributed within the city. The uniform travel demand can be used as the first approximation for the actual travel demand and serves as a basis for further analysis with more realistic travel demand. In fact, the uniform demand has frequently been used in continuous transportation models

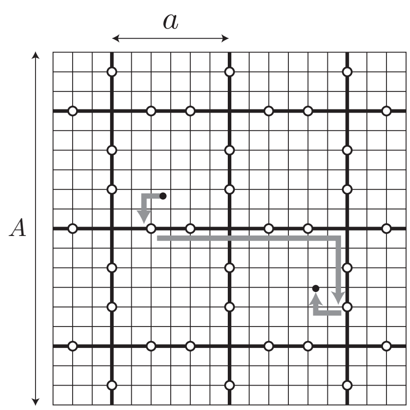

Figure 1: Grid road network in a square city

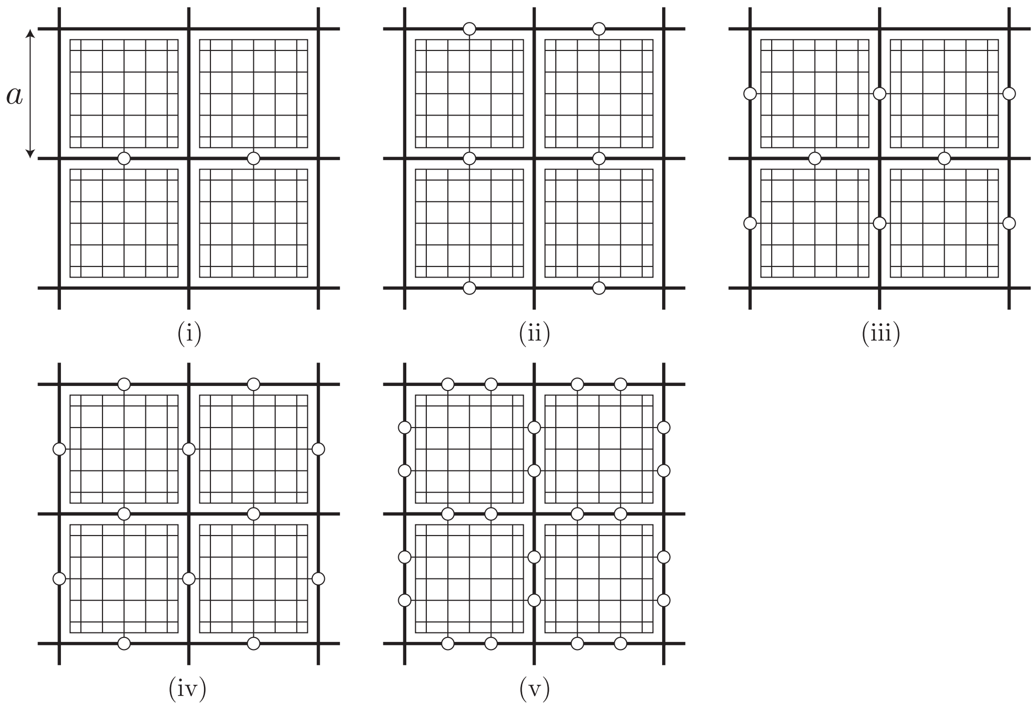

Figure 2: Patterns of intersections

[2, 5, 16]. More realistic travel demand can be incorporated by using spatial interaction models [19].

Every traveler uses both minor and major roads and follows the nearest intersection routing, as suggested by Miyagawa [14]. The movement of a traveler is shown in Figure 1. First, the traveler moves from origin along minor roads to the nearest intersection of minor and major roads. At the intersection, s/he transfers to major roads and moves to the intersection nearest to destination. Using again minor roads, s/he arrives at destination. Obviously, the travel time under the nearest intersection routing is not always the minimum travel time because whether or not travelers should use major roads depends on the location of origin and destination. Miyagawa [14], however, demonstrated that there exists a close relationship between the two travel time.

Our objective is to find the optimal pattern of intersections that connect minor and major roads. We focus on five patterns of intersections, as shown in Figure 2. In pattern (i), a intersection is located on the midpoint of a side of each square block. In patterns (ii), (iii), and (iv), a intersection is located on the midpoint of two, three, and four sides of each square block, respectively. In pattern (v), two intersections are located at equal intervals on four sides of each square block. These patterns are assumed to continue over the city. The density of intersections (number of intersections per unit length), denoted by λ, is

$$\lambda = \begin{cases} \frac{5}{4a} = \frac{5\Lambda}{8A^2}, & \text{(i)}, \\ \frac{3}{2a} = \frac{3\Lambda}{4A^2}, & \text{(ii)}, \\ \frac{7}{4a} = \frac{7\Lambda}{8A^2}, & \text{(iii)}, \\ \frac{2}{a} = \frac{\Lambda}{A^2}, & \text{(iv)}, \\ \frac{3}{a} = \frac{3\Lambda}{2A^2}, & \text{(v)}. \end{cases} \tag{2.1}$$

Note that λ includes not only intersections connecting minor and major roads but also intersections of two major roads. The average spacing of intersections is given by $1/\lambda$. We then evaluate the five intersection patterns from the average travel time. The theoretical results of these patterns will give an insight into empirical studies on actual patterns.

3. Average Travel Time

To obtain an analytical expression for the travel time, the travel distance on a grid network is approximated by the rectilinear distance on a continuous plane. The rectilinear distance between two points (x_1, y_1) and (x_2, y_2) is defined as $|x_1 - x_2| + |y_1 - y_2|$. The rectilinear distance is a good approximation for the actual travel distance in cities with a grid road network [1, 10, 12]. The accuracy of this continuous approximation was investigated by Miyagawa [14].

Let v_1 and v_2 be the travel speeds on minor and major roads, respectively. Both v_1 and v_2 represent free travel speeds and assumed to be constant irrespective of traffic volume. The average travel distance on minor roads is approximated by the average rectilinear distance from a uniformly distributed point in the square with side length a to the nearest intersection. The average rectilinear distance to the nearest intersection is (i) $3a/4$, (ii) $a/2$, (iii) $5a/12$, (iv) $a/3$, and (v) $43a/162$. For example, the average rectilinear distance for pattern (i) is

$$\frac{2}{a^2}\int_0^a\int_0^{a/2}(x+y)\,\mathrm{d}x\mathrm{d}y = \frac{3}{4}a, \tag{3.1}$$

where the intersection is at (0,0). The average travel time on minor roads, denoted by T_1, is then given by

$$T_1 = \begin{cases} \frac{3a}{4v_1} & = \frac{3A^2}{2v_1\Lambda}, \quad \text{(i)}, \\ \frac{a}{2v_1} & = \frac{A^2}{v_1\Lambda}, \quad \text{(ii)}, \\ \frac{5a}{12v_1} & = \frac{5A^2}{6v_1\Lambda}, \quad \text{(iii)}, \\ \frac{a}{3v_1} & = \frac{2A^2}{3v_1\Lambda}, \quad \text{(iv)}, \\ \frac{43a}{162v_1} & = \frac{43A^2}{81v_1\Lambda}, \quad \text{(v)}. \end{cases} \tag{3.2}$$

The average travel distance between intersections on major roads is approximated by the average rectilinear distance between two uniformly distributed points in the square with side length A. The average rectilinear distance is $2A/3$ [6]. The average travel time on major roads, denoted by T_2, is defined as the sum of the free travel time and the delay at intersections and given by

$$T_2 = \frac{2A}{3v_2} + \tau\lambda\frac{2A}{3}, \tag{3.3}$$

where τ is the delay at one intersection. The rationale for this expression was discussed by Koshizuka and Imai [11], who examined the relationship between the density of signalized intersections and the average travel time. Since every traveler uses minor roads twice (near origin and destination) and major roads once, the average travel time T is expressed as

$$T = 2T_1 + T_2 = \begin{cases} \frac{3A^2}{v_1\Lambda} + \frac{2A}{3v_2} + \frac{5\tau\Lambda}{12A}, & \text{(i)}, \\ \frac{2A^2}{v_1\Lambda} + \frac{2A}{3v_2} + \frac{\tau\Lambda}{2A}, & \text{(ii)}, \\ \frac{5A^2}{3v_1\Lambda} + \frac{2A}{3v_2} + \frac{7\tau\Lambda}{12A}, & \text{(iii)}, \\ \frac{4A^2}{3v_1\Lambda} + \frac{2A}{3v_2} + \frac{2\tau\Lambda}{3A}, & \text{(iv)}, \\ \frac{86A^2}{81v_1\Lambda} + \frac{2A}{3v_2} + \frac{\tau\Lambda}{A}, & \text{(v)}. \end{cases} \tag{3.4}$$

4. Optimal Intersection Pattern

The average travel time on minor and major roads for the five intersection patterns (i)–(v) are plotted in Figure 3, where A = 10km, Λ = 100km, v_1 = 20km/h, v_2 = 40km/h, τ = 0.5/60h. As the density of intersections increases, the travel time on minor roads T_1 decreases but the travel time on major roads T_2 increases. It follows that there exists a tradeoff between the travel time on minor and major roads.

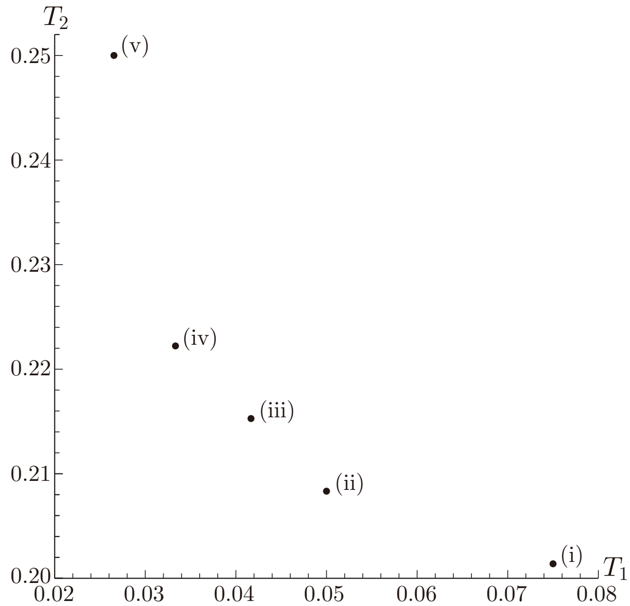

Figure 3: Average travel time on minor and major roads

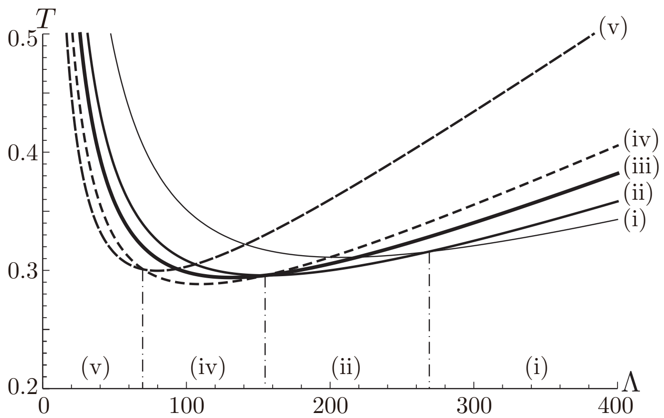

Figure 4: Length of major roads and average travel time

The average travel time is shown in Figure 4 as a function of length of major roads. The values of the other parameters are the same as those of Figure 3. It can be seen that as the road length increases, the average travel time first decreases and then increases. This is because the increase in the road length improves the accessibility to major roads but causes more intersection delay. The optimal intersection pattern that minimizes the average travel time is also shown in the figure. The optimal pattern varies according to the road length. If the city has a few major roads, pattern (v) is optimal. As the road length increases, patterns (iv), (ii), and (i) can be optimal. Thus, many intersections are required for cities with a few major roads to reduce the average travel time. On the other hand, a few intersections are sufficient for cities with many major roads. This result makes sense intuitively.

By comparing the average travel time (3.4), we can obtain the optimal intersection pattern and the condition that the pattern outperforms the others, as shown in Table 1. Note that the optimal density of intersections decreases with the road length Λ, the intersection delay τ, and the travel speed on minor roads v_1, and increases with the size of the city A. Note also that pattern (iii) cannot be optimal. The reason is that the average travel time for (iii) is always greater than that for either (ii) or (iv) (see Figure 4).

Table 1: Optimal intersection pattern

Pattern	Condition
(i)	$\Lambda > \frac{2\sqrt{3}A^{3/2}}{\sqrt{\tau v_1}}$
(ii)	$\frac{2A^{3/2}}{\sqrt{\tau v_1}} < \Lambda \le \frac{2\sqrt{3}A^{3/2}}{\sqrt{\tau v_1}}$
(iv)	$\frac{\sqrt{22}A^{3/2}}{3\sqrt{3\tau v_1}} < \Lambda \le \frac{2A^{3/2}}{\sqrt{\tau v_1}}$
(v)	$\Lambda \le \frac{\sqrt{22}A^{3/2}}{3\sqrt{3\tau v_1}}$

If the length of major roads is not given but a decision variable, we can obtain the optimal combination of the road length and intersection pattern. The length of major roads that minimizes the average travel time is

$$\Lambda^* = \begin{cases} \frac{6A^{3/2}}{\sqrt{5\tau v_1}}, & \text{(i)}, \\ \frac{2A^{3/2}}{\sqrt{\tau v_1}}, & \text{(ii)}, \\ \frac{\sqrt{2}A^{3/2}}{\sqrt{\tau v_1}}, & \text{(iv)}, \\ \frac{\sqrt{86}A^{3/2}}{9\sqrt{\tau v_1}}, & \text{(v)}. \end{cases} \tag{4.1}$$

Substituting into (3.4) yields

$$T^* = \begin{cases} \frac{\sqrt{5\tau A}}{\sqrt{v_1}} + \frac{2A}{3v_2}, & \text{(i)}, \\ \frac{2\sqrt{\tau A}}{\sqrt{v_1}} + \frac{2A}{3v_2}, & \text{(ii)}, \\ \frac{4\sqrt{2\tau A}}{3\sqrt{v_1}} + \frac{2A}{3v_2}, & \text{(iv)}, \\ \frac{2\sqrt{86\tau A}}{9\sqrt{v_1}} + \frac{2A}{3v_2}, & \text{(v)}. \end{cases} \tag{4.2}$$

Note that the average travel time for pattern (iv) is the minimum. That is, the optimal combination of the road length and intersection pattern is $\Lambda^* = \sqrt{2}A^{3/2}/\sqrt{\tau v_1}$ and pattern (iv). In fact, the average travel time is the minimum at $\Lambda = 109.5$ in Figure 4.

5. Application

As an application of the model, we consider the optimal intersection pattern for 23 special wards of Tokyo. The areas of the wards, the lengths of major roads (roads with width 5.5m or more), and the average spacings of intersections (intersections of two major roads and intersections of minor and major roads) are summarized in Table 2. The road network data were extracted from Digital Map 25000 (Spatial Data Framework) provided by Geospatial Information Authority of Japan. The square root of the area A and the road length Λ are plotted in Figure 5. Although the road length is approximately proportional to the size of the ward, it varies widely among wards. The optimal intersection pattern for $v_1 = 20$km/h, $\tau = 0.5/60$h obtained from Table 1 is also shown in the figure. The optimal pattern is expressed as the region on the A–Λ plane. It can be seen that pattern (i) is optimal for Adachi, Koto, and Minato, (ii) for Ota, Edogawa, and Itabashi, and (iv) for Setagaya, Nakano, and Meguro. Note that the optimal pattern depends not only on the road length but also on the size of wards. In fact, the optimal pattern for Minato is (i), though the road length of Minato is shorter than that of Ota, the optimal pattern for which is (ii). Note also that pattern (v) (and patterns with more intersections) cannot be optimal. Although these wards have neither a square form nor a complete grid network, the model at least can be used to evaluate the spacing of intersections. For example, the optimal pattern for Setagaya is (iv) and the optimal spacing is thus $A^2/\Lambda = 639.0$m, which is much greater than the actual spacing 60.4m. Obviously, the result depends on the travel speed on minor roads and the delay at intersections. They should be estimated from actual traffic data for more detailed evaluation.

Table 2: Area, length of major roads, and average intersection spacing of 23 wards of Tokyo

	Area [km^2]	Length [km]	Spacing [m]		Area [km^2]	Length [km]	Spacing [m]
Adachi	53.2	198.4	72.6	Nakano	15.6	30.6	53.5
Arakawa	10.2	26.7	65.4	Nerima	48.2	95.2	69.2
Bunkyo	11.3	40.1	61.4	Ota	59.5	139.8	79.4
Chiyoda	11.6	67.4	63.0	Setagaya	58.1	90.9	60.4
Chuo	10.2	69.9	60.2	Shibuya	15.1	50.0	59.4
Edogawa	49.9	139.1	75.7	Shinagawa	22.7	73.6	75.3
Itabashi	32.2	85.8	68.1	Shinjuku	18.2	69.5	53.3
Katsushika	34.8	67.1	67.9	Suginami	34.0	66.7	57.3
Kita	20.6	52.2	66.7	Sumida	13.8	53.8	67.1
Koto	39.5	174.1	92.1	Taito	10.1	50.0	61.1
Meguro	14.7	25.1	54.9	Toshima	13.0	33.2	49.0
Minato	20.3	100.7	76.6				

6. Conclusions

This paper has presented an analytical model for determining the optimal intersection pattern that minimizes the average travel time. The model explicitly considers the tradeoff between the accessibility to higher level roads and the delay at intersections, and thus supplies building blocks for analyzing the spacing of intersections in hierarchical road networks.

The analytical expression for the average travel time shows how the road length, the intersection delay, the travel speed, and the size of the city affect the optimal intersection pattern. Note that finding these relationships by using discrete network models requires computation of the average travel time for various combinations of the parameters. The relationships help planners to evaluate actual intersection patterns. For example, if the spacing of intersections of an actual road network is greater than that of the optimal pattern, consolidating some intersections should be considered to reduce the travel time. The model is also useful to determine the number and location of intersections to achieve a certain level of service.

The presented model focuses on the optimal intersection pattern, given the length of major roads. Future research should address the optimal structure of road networks and determine the length of major roads as well as the location of intersections. Comparison

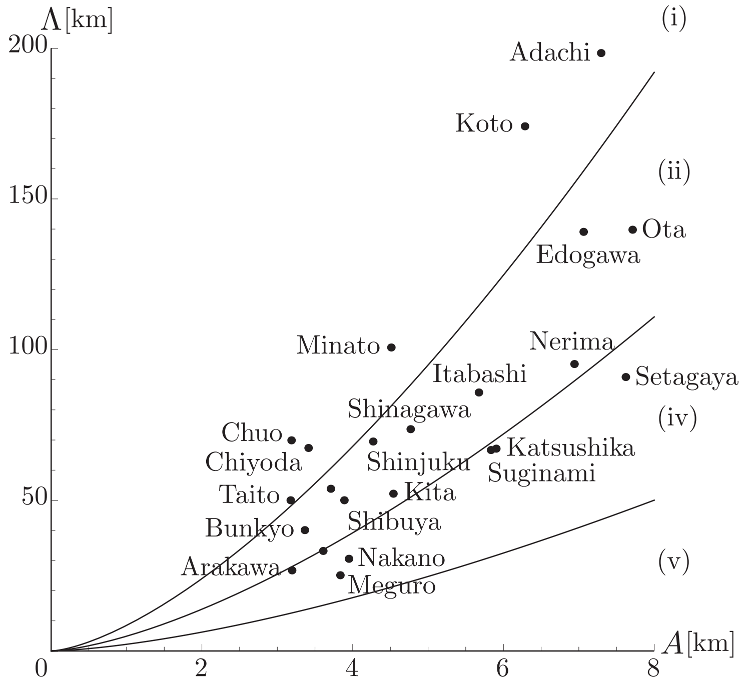

Figure 5: Optimal intersection pattern of 23 wards of Tokyo

with other types of networks such as a radial-arc network would also be interesting.

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Masashi Miyagawa
Department of Regional Social Management
University of Yamanashi
4-4-37 Takeda, Kofu
Yamanashi 400-8510, Japan
E-mail: mmiyagawa@yamanashi.ac.jp