An Empirical Examination of Intraday Volatility on Nikkei 225 Futures: A Bayesian Approach

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1. Introduction

In this paper, we analyze intraday volatilities of stock market in Japan. Intraday volatility have different features with daily volatility. It is widely known that intraday volatility have a U-shaped pattern during trading hours. In addition, intraday volatility is influenced by interday volatilities and macroeconomic announcements. Due to such complexities, we are confronted with some difficulties in parametric estimation of intraday volatility.

Stroud and Johannes [3] proposed a method by Bayesian modeling to estimate intraday volatility for high-frequency returns. They modeled intraday volatility by four components and estimated the parameters by sampling from posterior density using Markov Chain Monte Carlo (MCMC) methods.

We introduce Stroud and Johannes [3], apply the method for five-minute returns of Nikkei 225 Futures and report the empirical results.

2. Stochastic volatility models

We consider a Stochastic Volatility (SV) model for intraday financial returns. Intraday logarithmic price returns are described by

\[ y_t = 100 \cdot \log \left( \frac{P_t}{P_{t-1}} \right) = \mu + v_t \varepsilon_t + J_t Z_t^y \quad (1) \]

for \( t = 1, 2, \ldots, T \) where \( P_t \) is a future price, \( \mu \) is a mean return, \( v_t \) is a volatility, \( \varepsilon_t = \sqrt{X_t} \varepsilon_t \), where \( \varepsilon_t \sim i.i.d \mathcal{N}(0, 1) \) and \( \lambda_t \sim i.i.d \mathcal{IG} \left( \frac{\gamma}{2}, \frac{\gamma}{2} \right) \), \( J_t \) is a jump indicator with \( P[J_t = 1] = \kappa \), and \( Z_t^y \) is a return jump, where \( Z_t^y \sim i.i.d \mathcal{N}(\mu_y, \sigma_y^2) \). Here, \( \mathcal{N} \) is a normal distribution and \( \mathcal{IG} \) is an inverse gamma distribution. The volatilities, \( v_t \), are expressed as a multiplicative form:

\[ v_t = \sigma \cdot X_{t,1} \cdot X_{t,2} \cdot S_t \cdot A_t \quad (2) \]

where \( X_{t,1} \) and \( X_{t,2} \) are SV processes, \( S_t \) is seasonal component, and \( A_t \) is announcement component. Then, the logarithm of the variance can be rewritten as

\[ h_t = \log(v_t^2) = \mu_h + x_{t,1} + x_{t,2} + s_t + a_t \quad (3) \]

where \( \mu_h = \log(\sigma^2) \), \( x_{t,i} = \log(X_{t,i}^2) \) for \( i = 1, 2 \), \( s_t = \log(S_t^2) \), and \( a_t = \log(A_t^2) \).

In what follows, we explain each component in (3). The log-volatilities, \( x_{t,1} \) and \( x_{t,2} \), are given by

\[ x_{t+1,1} = \phi_1 x_{t,1} + \sigma_1 \eta_{t,1}, \quad x_{t+1,2} = \phi_2 x_{t,2} + \sigma_2 \eta_{t,2} + J_t Z_t^y \quad (4, 5) \]

where \( \eta_{t,i} \sim i.i.d \mathcal{N}(0, 1) \) for \( i = 1, 2 \), \( \rho = corr(\varepsilon_t, \varepsilon_{t+1}) \) represents the correlation between returns and the log-volatility \( x_{t,2} \), \( J_t \) is a jump time, and \( Z_t^y \sim i.i.d \mathcal{N}(\mu_y, \sigma_y^2) \) is a jump size in the log-volatility \( x_{t,2} \). The SV process \( X_{t,1} \) explains the persistence of interday volatility. On the other hand, the SV process \( X_{t,2} \) represents the short-term impact of high-frequency news or liquidity events. We refer to \( X_{t,1} \) and \( X_{t,2} \) as “slow” and “fast” volatility factors, respectively, and assume that \( 0 < \phi_2 < \phi_1 < 1 \).

Seasonal components explain the deterministic volatility patterns during trading hours. It is known that the volatility patterns typically have the smooth U-shaped patterns. The seasonal components, \( s_t \), are given by

\[ s_t = \sum_{k=1}^{K} H_{tk} \beta_k \quad (6) \]

where \( H_{tk} \) is an indicator, and \( \beta_k \) is the seasonal effect at period \( k \). The coefficients \( \beta_k \) for seasonal components are estimated using state-space form for cubic smoothing splines.

Announcement components are factors for the effect of macroeconomic announcements and specific events. The announcement components are

\[ a_t = \sum_{i=1}^{n} \sum_{k=1}^{5} I_{ik} \alpha_{ik} \quad (7) \]
where $\alpha_{ik}$ is the announcement effect for news type $i$ at $k$ period after the news release for $i = 1, 2, \ldots, n$ and $k = 1, 2, \ldots, 5$, and $I_{ikk}$ is an indicator for news type $i$ at period $k$. The coefficients $\alpha_{ik}$ for announcement components are modeled by a state-space form as with seasonal components.

3. Estimation approach

The logarithmic price return equation (1) can be written as

$$y_t^* = h_t + \zeta_t$$

where

$$y_t^* = \log \left( \frac{(y_t - \mu - J_t Z_t^y)^2}{\lambda_t} \right),$$
$$\zeta_t = \log(\zeta_t^2),$$
$$d_t = \text{sign}(y_t - \mu - J_t Z_t^y)$$

and we let $y^* = (y_1^*, \ldots, y_T^*)'$. We approximate the distribution of $\zeta_t$ by a mixture of normal distributions

$$f(\zeta_t) = \sum_{j=1}^{K} p_j N(\zeta_t | m_j, v^2_j)$$

where $N(\zeta_t | m_j, v^2_j)$ is a density function of normal distribution with mean $m_j$ and variance $v^2_j$. The conditional distribution of $\eta_{t,2}$ given $d_t, \zeta_t$ and $\rho$ is given by

$$\eta_{t,2} | d_t, \zeta_t, \rho \sim N(d_t \rho \exp \left( \frac{\zeta_t^2}{2} \right), 1 - \rho^2)$$

We then approximate the joint distribution of $\zeta_t$ and $\eta_{t,2}$ by a following mixture of normal distributions

$$p(\zeta_t, \eta_{t,2} | d_t, \rho) = \sum_{j=1}^{K} p_j N(\zeta_t | m_j, v^2_j)$$
$$\times N(\eta_{t,2} | d_t \rho (a_j^* + b_j^* \zeta_t), 1 - \rho^2)$$

where $(p_j, m_j, v_j, a_j^*, b_j^*)$ are constants shown in Omori et al. [2]. We introduce the latent mixture component indicators $\omega_{t} \in \{1, 2, \ldots, K\}$ and let $\omega = (\omega_1, \ldots, \omega_T)$.

Under the above setup, the joint posterior density for the model is given by

$$p(\mathbf{x}, \lambda, \mathbf{J}, \mathbf{Z}, \beta, \alpha, \theta | y)$$
$$\propto p(y | \mathbf{x}, \lambda, \mathbf{J}, \mathbf{Z}, \beta, \alpha, \theta)$$
$$p(\mathbf{x}, \lambda, \mathbf{J}, \mathbf{Z} | \theta)p(\beta | \theta)p(\alpha | \theta)p(\theta)$$

where $\mathbf{x} = (x_1, \ldots, x_T)$, $\mathbf{x}_t = (x_{t,1}, x_{t,2})$, $\lambda = (\lambda_1, \ldots, \lambda_T)$, $\mathbf{J} = (J_1, \ldots, J_T)$, $\mathbf{Z} = (Z_1, \ldots, Z_T)$, $\mathbf{Z}_t = (Z^y_t, Z^c_t)$, $\beta = (\beta_1, \ldots, \beta_K)'$, $\alpha = (\alpha_1, \ldots, \alpha_n)'$, $\alpha_k = (\alpha_1, \ldots, \alpha_3)$, $\theta = (\mu_h, \phi_1, \phi_2, \sigma_1, \sigma_2, \rho, \nu, \kappa, \mu_y, \sigma_y, \mu_v, \sigma_v, \tau_a)$ are parameters, and $y = (y_1, \ldots, y_T)'$ are data of returns.

The states and parameters for the model are estimated using MCMC algorithm.

4. Estimation results

We estimate intraday volatilities for five-minute returns of Nikkei 225 Futures by applying Stroud and Johannes’s [3] method. The empirical results will be reported at the conference.

References

