The effects of asset liquidity on dynamic bankruptcy decisions

1. Introduction

During financial distress, a firm can be acquired by other parties as an alternative to bankruptcy. However, a firm does not always meet other parties that offer favorable bid price. Even if a firm tries to sell total or partial assets, it sometimes fails and results in bankruptcy. Previous papers do not focus on this illiquidity associated with asset sale opportunities. This research incorporates the asset illiquidity into the standard structural model and reveals its effects on the bankruptcy process.

2. Model Setup

Consider a firm that is receiving EBIT (earnings before interests and taxes) \( X(t) \), where \( X(t) \) follows a geometric Brownian motion

\[
dX(t) = \mu X(t)dt + \sigma X(t)dB(t) \quad (t > 0), \quad X(0) = x.
\]

We assume that the firm has already issued console debt with coupon \( C \) and that \( X(0) = x \) is sufficiently large to exclude the firm’s bankruptcy at time 0. A positive constant \( r \) denotes the interest rate, and for convergence we assume that \( r > \mu \). We assume the corporate tax rate \( \tau \).

As in [1], [2], and [3], we examine the firm’s bankruptcy choice between selling out and default. In the sales case, the firm sells the total assets and receives the sales value from the bidder. Following the absolutely priority rule, the debt holders are repaid the face value of debt, \( C/r \). The shareholders receive the remains. Most importantly, we model the illiquidity of asset sales as follows. Following a Poisson process with the arrival rate \( \lambda \), potential acquirers arrive and bid \( P(X(t)) = aX(t)/(r - \mu) + \theta \) for the firm, where \( a \in [0, 1], \theta > 0 \) are constants. If the firm rejects an acquirer’s offer, the firm need to wait until another acquirer arrives. In the default case, the shareholders stop paying the coupon \( C \), and the former debt holders take over the firm. A fraction \( \alpha \in (0, 1) \) of the firm value is lost to default costs.

3. Unlevered Firm

We define, for \( y > 0 \),

\[
\beta(y) = 0.5 - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - 0.5\right)^2 + \frac{2(r + y)}{\sigma^2}} (> 1),
\]

\[
\gamma(y) = 0.5 - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} - 0.5\right)^2 + \frac{2(r + y)}{\sigma^2}} (< 0).
\]

For simplicity, we denote \( \beta(0) \) and \( \gamma(0) \) by \( \beta \) and \( \gamma \), respectively. Consider a threshold policy inf\( \{t \geq 0 \mid X(t) \leq x_U^*\} \). The unlevered firm value \( E_U(x) \) satisfies

\[
\mu x E_U'(x) + 0.5 \sigma^2 x^2 E_U''(x) + \lambda(1 - \tau) P(x) - E_U(x) + (1 - \tau) x = r E_U(x),
\]

\[
(1 - \tau) x = r E_U(x), \quad (x < x_U^*)
\]

\[
(1 - \tau) \quad \left\{ \begin{array}{l}
\frac{x}{r - \mu} + \left( \frac{x}{x_U^*} \right)^\gamma \left( \frac{(a - 1)x_U^*}{r - \mu} + \theta \right) \\
(x > x_U^*)
\end{array} \right.
\]

where the sell-out threshold \( x_U^* \) is defined by

\[
x_U^* = \frac{(r + \lambda)(\gamma - \beta(\lambda))r + \gamma \lambda}{(r + \lambda)((\gamma - \beta(\lambda))(r - \mu) + (\gamma - 1)\lambda)} - 1 - a.
\]

With higher asset liquidity, the solution approaches the standard solution, whereas with lower asset liquidity, the solution approaches the NPV solution.
4. Levered Firm

We define
\[ \epsilon = (1 - a) \frac{\lambda}{\max} \in (0, 1], \]
\[ \theta = \theta - \frac{\tau C}{(1 - \tau)r}. \]

The solutions are classified into the four cases.

**Proposition 2**

**Case (S):**

\[ \frac{C}{r} \theta \leq \frac{(1 - \tau)\lambda}{\tau + (1 - \tau)(r + \lambda)}. \]

The firm prefers to sell out.

**Case (D):**

The firm prefers to default.

\[ E(x) = \begin{cases} 
0 & (x \leq x^d), \\
(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + A_1x^\beta + A_2x^\gamma \right\} & (x \in (x^d, x^s]), \\
(1 - \tau) \left\{ \frac{(r + a\lambda - \mu)x}{(r - \mu)(r + \lambda - \mu)} - \frac{C}{r} + \frac{\lambda \hat{\theta}}{r + \lambda} + A_3x^\beta(\alpha) + A_4x^\gamma(\alpha) \right\} & (x \in [x^s, x^2]), \\
(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x^2} \right)^\gamma \left( \frac{(a - 1)x^s}{r - \mu} + \hat{\theta} \right) \right\} & (x > x^s), \\
\end{cases} \]

**Case (DOS):**

\[ E(x) \begin{cases} 
0 & (x \leq x^d), \\
(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + A_1x^\beta + A_2x^\gamma \right\} & (x \in (x^d, x^s]), \\
(1 - \tau) \left\{ \frac{(r + a\lambda - \mu)x}{(r - \mu)(r + \lambda - \mu)} - \frac{C}{r} + \frac{\lambda \hat{\theta}}{r + \lambda} + A_3x^\beta(\alpha) + A_4x^\gamma(\alpha) \right\} & (x \in [x^s, x^2]), \\
(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x^2} \right)^\gamma \left( \frac{(a - 1)x^s}{r - \mu} + \hat{\theta} \right) \right\} & (x > x^s), \\
\end{cases} \]

**Cases (DS) or (DOS):**

\[ \frac{(1 - \tau)\lambda}{\tau + (1 - \tau)(r + \lambda)} \leq \frac{1 - \tau}{\tau + (1 - \tau)e}. \]

The firm can either defaults or sells out.

**Case (DS):**

\[ E(x) \begin{cases} 
0 & (x \leq x^d), \\
(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + A_1x^\beta + A_2x^\gamma \right\} & (x \in (x^d, x^s]), \\
(1 - \tau) \left\{ \frac{x}{r - \mu} - \frac{C}{r} + \left( \frac{x}{x^s} \right)^\gamma \left( \frac{(a - 1)x^s}{r - \mu} + \hat{\theta} \right) \right\} & (x > x^s), \\
\end{cases} \]

Similarly, we have analytical expressions of debt values and default probabilities. In the presentation, we also provide numerical examples and implications.

**References**

