

# Strategic design of station-based one-way carsharing systems with consideration of risk aversion

05001233 筑波大学大学院システム情報工学研究科 \*張 凱 ZHANG Kai  
 01309510 筑波大学システム情報系 高野祐一 TAKANO Yuichi  
 05001183 筑波大学大学院システム情報工学研究科 汪 玉柱 WANG Yuzhu  
 01703540 筑波大学システム情報系 吉瀬章子 YOSHISE Akiko

## 1. Introduction

Carsharing is receiving increasing attention as an alternative to private vehicle ownership. In the context of Society 5.0, Toyota launched its own station-based one-way carsharing system “Ha:mo RIDE” in 2012. In such a system, users are allowed to drop off at any stations with great flexibility, which is different from the round-trip system requiring users to return cars to their original sites.

In this study, we focus on the strategic design of station-based one-way carsharing systems and determine jointly the location and capacity of stations and the fleet size considering risk aversion. A two-stage risk-averse stochastic programming model is proposed to maximize the mean return and minimize the risk, where the conditional value-at-risk (CVaR) is specified as the risk measure. Compared to the traditional risk-neutral models [1] using the expectation value as the preference criterion, our model can provide more robust solutions under certain realizations of random data. In order to solve our model efficiently, we also develop two algorithms, namely the branch-and-cut (B&C) algorithm and the scenario decomposition (SD) algorithm. Based on the data from Ha:mo RIDE Toyota, we conduct computational experiments to verify the model and algorithms.

## 2. Formulation

### 2.1. General mean-CVaR model

CVaR is a downside risk measure to quantify tail losses, which was first proposed by Rockafellar

and Uryasev [2]. Unlike the traditional risk measure value-at-risk (VaR), CVaR has more attractive mathematical properties (e.g. subadditivity, convexity).

In practical application, the following mean-CVaR model [3, 4] is usually considered to minimize the risk and maximize the return (or minimize the loss) in the decision-making process.

$$\min_{\mathbf{x}, \alpha} \{ \lambda \mathbb{E}[\mathcal{L}(\mathbf{x}, \mathbf{y})] + (1 - \lambda) \mathcal{F}_\beta(\mathbf{x}, \alpha) \} \quad (1)$$

where  $\lambda \in [0, 1]$  is a weight value;  $\mathbb{E}[\cdot]$  is the expectation function;  $\mathcal{L}(\mathbf{x}, \mathbf{y})$  is a loss function with respect to decision vector  $\mathbf{x} \in \mathbb{R}^n$  and uncertain vector  $\mathbf{y} \in \mathbb{R}^m$ ;  $\mathcal{F}_\beta(\mathbf{x}, \alpha)$  is the auxiliary function [2] to calculate CVaR, which can be written by

$$\begin{aligned} \mathcal{F}_\beta(\mathbf{x}, \alpha) &= \alpha + \frac{1}{1 - \beta} \int_{\mathbf{y} \in \mathbb{R}^m} [\mathcal{L}(\mathbf{x}, \mathbf{y}) - \alpha]_+ p(\mathbf{y}) d\mathbf{y} \\ &\approx \alpha + \frac{1}{(1 - \beta)|S|} \sum_{s \in S} [\mathcal{L}(\mathbf{x}, \mathbf{y}^s) - \alpha]_+ \end{aligned} \quad (2)$$

where  $\beta$  is the confidence level;  $\alpha \in \mathbb{R}$  is an auxiliary variable;  $[\cdot]_+ = \max\{\cdot, 0\}$ ;  $p(\mathbf{y})$  is the probability density function associated with  $\mathbf{y}$ ;  $\mathbf{y}^s$  is the realization of  $\mathbf{y}$  for  $s$  in the scenario set  $S$ .

### 2.2. Strategic model

We consider applying the mean-CVaR model to carsharing systems and give an outline of our model below instead of a detailed version due to the limited space.

#### Objective function

- Mean-CVaR function (to be minimized)  
(Loss = Operating costs – Trip revenue)

#### Constraints

1. Budget constraint  
(Station and vehicle costs)
2. Maximum capacity constraint
3. Flow conservation
4. Enough vehicles available at origin stations
5. Enough parking spaces available at destination stations
6. Fleet size constraint

#### Decision variables

1. Number of parking spaces
2. Fleet size
3. Auxiliary variable for CVaR calculation
4. Number of vehicles at each site at each time interval in each scenario
5. Binary variables, whether to serve a trip

### 3. Solution methods

Due to the CVaR function being nonlinear and nondifferentiable, the strategic decision problem becomes more difficult as the number of scenarios increases. To deal with the problem efficiently, we develop two solution methods, that is, the B&C algorithm and the SD algorithm. The former algorithm mainly focuses on the nonlinear CVaR function, while the latter one pays attention to the special block-angular structure of the stochastic problem.

The B&C algorithm can handle the CVaR function efficiently, the problem size still depends on the number of scenarios. As a result, the algorithm may have difficulty solving the problem involving many scenarios. The SD algorithm can compensate for the disadvantage by using a decomposition technique. The detailed algorithms will be explained in the presentation.

## 4. Computational experiments

### 4.1. Parameter settings

There were 55 stations with different capacities in the Ha:mo RIDE system. The sites of these stations were assumed to be the potential location sites, and the current station capacities were regarded as upper limits on the number of parking

spaces. We generated the demand scenarios by using a Poisson distribution and the user history. Each trip is a tuple of four elements: origin, destination, departure time, and arrival time. The price of each trip can be computed with the information above. Values of the remaining parameters were appropriately determined according to the survey.

### 4.2. Computational results

The main findings are as follows.

1. By solving the problems with different weight values, we obtained the efficient frontiers, which revealed a positive correlation between return and CVaR.
2. It is better to consider both the return and risk in the objective function so that the strategic decisions may lead to a higher out-of-sample return.
3. The B&C algorithm and the SD method are effective at solving small- and medium-scale problems. Furthermore, The scenario decomposition can cope with large-scale problems that cannot be solved by the direct usage of solver or by the B&C algorithm.

## References

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