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A Note on Sensitivity Analysis of Epistemic Uncertainty in Fault Trees

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1. Introduction

Fault tree (FT) plays an important role in reliability engineering, and represents a correlation between component failures and system failure. FT provides some quantitative measures for the system reliability/availability from component reliabilities/availabilities. By using such quantitative measures, we can design highly-reliable system in development phase. However, since the system reliability measure is directly computed from reliabilities of components, uncertainty of component reliability causes the uncertainty of system reliability. More precisely, if we did not estimate component reliability well, the resulting estimate of system reliability is far from the true system reliability. This is known as a problem on epistemic uncertainty. The statistical errors on parameter estimation are regarded as the epistemic uncertainty. The paper [1] discussed the method to correct a bias of the system reliability evaluation by considering the epistemic uncertainty of model parameters in the context of continuous-time Markov chains (CTMCs). On the other hand, the paper [2] presented a variance-based sensitivity analysis of epistemic uncertainty in CTMCs. The idea is similar to the analysis of variance which is used in statistics, and it provides how to determine what parameters should be estimated more accurately to mitigate the epistemic uncertainty of system reliability. In this paper, we present the variance-based sensitivity analysis of epistemic uncertainty in FT.

2. Fault Tree

Consider a FT to represent the relationship between system and component failures. FT consists of two classes of events; basic and composite events. Each event has two states; 0 and 1. The basic event usually represents the component is failure or not. If the state is 1, the corresponding component is failed. Otherwise, if the state is 0, the component works. Composite states are generated by two or more events with one gate. The gate means the casual relationship of the composite event and the other events. The AND and OR gates are typical ones in the fundamental FT. When the composite event is generated by an AND gate, the state of composite event becomes 1 only when the states of all the other events connected to the AND gate are 1. In usual, the basic events are placed on the bottom, and the composite event is placed on the top of the other events. Then the FT provides a tree structure and the top event is the event for the system failure.

The FT can also be represented by a structure function. Let $F(x_1, \dots, x_n)$ be a structure function where $x_1, \dots, x_n$ are the states of basic events. Then $F(x_1, \dots, x_n)$ means the state of top event. In the structure function, AND and OR gates are expressed by $x_1 x_2$ and $1-(1-x_1)(1-x_2)$, respectively. Note that this is slightly different from a simple boolean function in the Boolean algebra. By using such expression, the system availability A_s can be obtained by $A_s = F(A_1, \dots, A_n)$ where $A_1, \dots, A_n$ are component availabilities. Also we have the following recursive formula (Shannon decomposition) for the structure function:

$$F(x_1, \dots, x_n) = x_1 F(1, x_2, \dots, x_n) + (1-x_1) F(0, x_2, \dots, x_n). \quad (1)$$

3. Epistemic Uncertainty Analysis

Here we consider the epistemic uncertainty of system availability in FT and its sensitivity analysis. Let $A_1(\boldsymbol{\theta}_1), \dots A_n(\boldsymbol{\theta}_n)$ be component availabilities with parameter vectors $\boldsymbol{\theta}_1, \dots \boldsymbol{\theta}_n$ for component 1 though n. Then the system availability becomes $A_s(\boldsymbol{\theta}_s)$ where $\boldsymbol{\theta}_s = (\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_n)$.

The concept of epistemic uncertainty is to regard parameters as random variables. When $\boldsymbol{\Theta}_i$ is defined by a random variable vector for the parameter vector $\boldsymbol{\theta}_i$, we compute the expected system availability $\mathrm{E}[A(\boldsymbol{\Theta}_s)]$ in the context of epistemic uncertainty analysis. On the other hand, the variance-based sensitivity for epistemic uncertainty focuses on the fraction of variances, i.e., the sensitivity of parameter θ_i is defined by

$$S_i = \frac{\mathrm{Var}[\mathrm{E}[A_s(\boldsymbol{\Theta}_s)|\Theta_i]]}{\mathrm{Var}[A_s(\boldsymbol{\Theta}_s)]}, \quad (2)$$

where Θ_i is a random variable for the parameter θ_i. The above is similar to the contribution in ANOVA.

In [2], they provided the approximation for the above variances with the first two derivatives. For example, the variance of system availability can be computed as

$$\mathrm{Var}[A_s(\boldsymbol{\Theta}_s)] \approx \sum_{l=1}^{L} \left(\left. \frac{\partial A_s(\boldsymbol{\theta}_s)}{\partial \theta_l} \right|_{\boldsymbol{\theta}_s = \mathrm{E}[\boldsymbol{\Theta}_s]} \right)^2 \mathrm{Var}[\Theta_l] - \frac{1}{4} \left(\sum_{l=1}^{L} \left. \frac{\partial^2 A_s(\boldsymbol{\theta}_s)}{\partial \theta_l^2} \right|_{\boldsymbol{\theta}_s = \mathrm{E}[\boldsymbol{\Theta}_s]} \mathrm{Var}[\Theta_l] \right)^2. \quad (3)$$

For the sake of simplicity, we define $\boldsymbol{\theta}_s = (\theta_1, \dots, \theta_L)$. In the above, we need the first two derivative of system availability with respect to a model parameter. In the case of FT, the first two derivative of system availability can be computed by applying the automatic differentiation to Eq. (1).

4. Numerical Illustration

Consider a simple computer system composed of hardware and software components; CPU, memory (Mem), power subsystem (Pow), network device (Net), cooling subsystem (Cool) and operating system (OS), whose FT is depicted in Fig. 1. Then the system availability can be computed as $A_S = A_{OS} \prod_{i \in \mathrm{HW}} A_i$ where $\mathrm{HW} = \{\mathrm{CPU, Mem, Pow, Net, Cool}\}$. Also the component availabilities are $A_i = \mu_i/(\mu_i + \lambda_i)$ in which λ_i and μ_i are the failure rate and repair rate of component i. They are set as $\lambda_{CPU} = 1/2,50,000$, $\lambda_{Mem} = 1/480,000$, $\lambda_{Pow} = 1/670,000$, $\lambda_{Net} = 1/120,000$, $\lambda_{Cool} = 1/3,100,000$, $\lambda_{OS} = 1/1,440$, $\mu_{CPU} = \mu_{Mem} = \mu_{Pow} = \mu_{Net} = \mu_{Cool} = 1/0.5$ and $\mu_{OS} = 1$. Also, the uncertainty of model parameters is determined so that the coefficient of variation (CV) is 0.2.

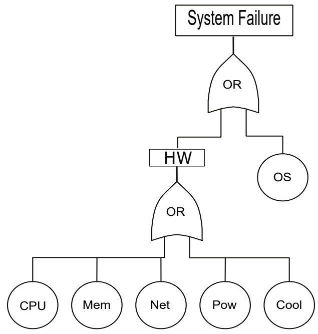

Figure 1: FT of computer system.

Table 1: Sensitivities for epistemic uncertainty of system availability.

Component	S_{λ_i}	S_{μ_i}
CPU	4.2388e-8	4.0693e-8
Mem	1.1498e-6	1.1039e-6
Net	5.9017e-7	5.6657e-7
Pow	1.8397e-5	1.7662e-5
Cool	2.7568e-8	2.6465e-8
OS	5.1034e-1	4.8995e-1

Table 1 shows the sensitivity for epistemic uncertainty of system availability. From the table, the effects on the system availability of uncertainty of failure and repair are almost same. On the other hand, the uncertainty of OS parameters strongly affects the uncertainty of system availability. To mitigate the uncertainty of system availability, it is effective to reduce the uncertainty of OS parameters.

References

[1] H. Okamura, T. Dohi and K.S. Trivedi, "Parametric uncertainty propagation through dependability models," In: Proc. 2018 8th Latin-American Symposium on Dependable Computing (LADC '18), IEEE, pp. 10–18, 2018.

[2] J. Zhang, J. Zheng, H. Okamura and T. Dohi, "A Note on Variance-Based Sensitivity Analysis for Continuous-Time Markov Chains Based on Moment Approximation," IEICE Tech. Rep., vol. 120, no. 286, R2020-33, pp. 18-23, 2020.